AP Physics C: Mechanics
Unit 7 – Simple Harmonic Motion and Oscillations
Assessment

Name: ____ Period: _________

Summary

- **Hooke's Law** – The force applied to or supplied by a spring:

$$F_x^{Applied} = kx$$

$$F_x^{Restorative} = -kx$$

- **Oscillatory Motion** is periodic motion that varies with time about an equilibrium position.
- **Simple Harmonic Motion** is defined as oscillatory motion under the influence of a retarding force (therefore, acceleration) that is proportional to the amount of displacement from an equilibrium position. Simple Harmonic Motion is Oscillatory Motion with the following conditions:
    1. The object exhibits sinusoidal behavior.
    2. The object is acting under the influence of a restorative force.
    3. The motion is proportional to the initial amount of displacement from the equilibrium condition.
- The **position** (displacement if unit vectors are utilized) of a simple harmonic oscillator is given by:

$$x = Acos(\omega t + \delta)$$

where: A is the amount of the motion.
Ω is the angular frequency (rad/s).
δ is the phase constant

- The time (in seconds) for one complete vibration, or oscillation, is called the **period** of the motion, *T*, defined by:

$$T = \frac{2\pi}{\omega}$$

- The number of cycles that happen in one second is called the **frequency** of the motion, *f*, measured in cycles/sec or Hertz (Hz). Frequency is the number of oscillations (less than, equal to, or greater than 1) in exactly 1 second; the period is the time it takes for one oscillation to occur. Frequency is the inverse of the period:

$$f = \frac{1}{T}$$

- Because a simple harmonic oscillatory does not undergo constant acceleration, the 4 equations of kinematics do not apply. From the expression for position, they are:

$$v = \frac{dx}{dt} = -\omega Asin(\omega t + \delta)$$

$$a = \frac{dv}{dt} - \omega^2 Acos(\omega t + \delta)$$

Note: Maximum velocity is ωA.
Maximum acceleration is $\omega^2 A$.
Velocity is zero at the turning points, $\pm A$.
Speed is a maximum at the equilibrium point, $x = 0$.
The magnitude of the acceleration is greatest at the turning points.
The magnitude of the acceleration is zero at the equilibrium point, $x = 0$.

- A mass-spring system exhibits simple harmonic motion on a frictionless surface has a period given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

- The **kinetic energy** of a simple harmonic oscillator is given by:

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 sin^2(\omega t + \delta)$$

Note: Kinetic energy is a maximum at the equilibrium position.
Kinematic energy is 0 at the turning points.

- The **potential energy** of a simple harmonic oscillator is given by:

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 cos^2(\omega t + \delta)$$

Note: Potential energy is a maximum at the turning points.
Potential energy is 0 at the equilibrium point.

- In the absent of nonconservative forces, the **total energy** of a simple harmonic oscillator is constant and given by:

$$E = \frac{1}{2}kA^2$$

- A **simple pendulum** of length *L* exhibits simple harmonic motion for small angular displacements from the vertical with a **period** given by (note that *T* is not a function of mass; that is, independent of the suspended mass, called a **bob**):

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- A **physical pendulum** exhibits simple harmonic motion about a pivot that does not go through the center of mass and has a period and frequency given by:

$$T = 2\pi\sqrt{\frac{I}{mgd}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

- A **damped oscillation** occurs where there is the presence of a dissipative or nonconservative force in the system, causing the mechanical energy to decrease with time and ultimately bring the system to rest, often at the equilibrium point (but it can be at another point if the static frictional force is greater than the restorative force). Damping can be compensated for with an external periodic driver. If the periodic driver matches the natural frequency of the oscillator when it is undamped, continually adding energy into the system, the amplitude increases without limit (reality: until the mechanical limit is overwhelmed and it breaks or shatters).

Concepts

1) Determine whether or not the following quantities can be in the same direction for simple harmonic motion. Explain your reasoning.
   a. Displacement and velocity.
   b. Velocity and acceleration.
   c. Displacement and acceleration.

2) Give an example of a damped oscillation that is commonly observed.

3) What is the total distance (!) travelled by a body executing simple harmonic motion if its amplitude is *A*?

4) If the position of a particle is given by $x = -A\cos(\omega t)$, what is the phase angle δ?

5) Does the displacement of an oscillating particle between $t = 0$ s and a later time t necessarily equal the position of the particle at time t? Explain.

6) If a mass-spring system is hung vertically and set into oscillation, why does its motion eventually stop? What is this called?

7) Why can neither the kinetic energy nor the potential energy of a mass-spring system never be negative?

8) A mass-spring system undergoes simple harmonic motion with an amplitude *A*. Does the total energy change if the mass is doubled but the amplitude is not changed? Do the potential and kinetic energies depend on mass? Explain.

9) A simple pendulum is suspended from the ceiling of a stationary elevator and the period is determined. Describe the changes, if any, in the period if the elevator

a) accelerates upward,
b) accelerates downward, and
c) moves with constant velocity.

Problems

10) The displacement of a body undergoing simple harmonic motion is given by the expression

$$x = (8.0\ cm)cos\left(2t + \frac{\pi}{3}\right)$$

a) Calculate the velocity and acceleration at $t = \pi/2$ s.
b) Determine the maximum speed and the time ($t > 0$) at which the particle has this speed.
c) Determine the maximum acceleration and the earliest time ($t > 0$) at which the particle has this acceleration.

11) A 0.5 kg mass attached to a spring of force constant 8 N/m vibrates with simple harmonic motion with an amplitude of 10 cm. Calculate

a) the maximum value of its speed and acceleration,
b) the speed and acceleration when the mass is at $x = 6$ cm from the equilibrium position, and
c) the time it takes the mass to move from $x = 0$ to $x = 8$ cm.

12) A particle executes simple harmonic motion with an amplitude of 3.0 cm. At what displacement from the midpoint of its motion will its speed equal one half of its maximum speed?

13) A physical pendulum has the characteristics of being planar (that is, it exists in the *xy*-plane and you may ignore its dimension along the *z*-axis) and exhibits simple harmonic motion. Its mass is 2.2 kg and the measured frequency is 1.5 Hz. The pivot of this physical pendulum is located 0.35 m from the center of mass. Determine the moment of inertia for this pendulum.

14) Consider a simple pendulum.

a) Calculate the total mechanical energy of the pendulum as a function of speed, v, and angle θ.
b) Show that when $\theta << 0$, the potential energy can be expressed as

$$\frac{1}{2}mgL\theta^2 = \frac{1}{2}m\omega^2 s^2$$

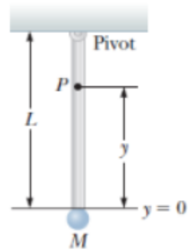
Note 1: When asked to show or derive, you must represent each step and begin with fundamental principles.

Note 2: You may use the approximation $cos\theta = 1 - \frac{\theta^2}{2}$ for small θ. Such a hint would be given by the College Board on an AP test.

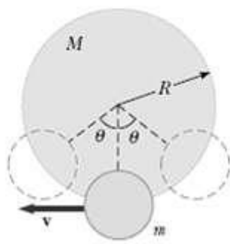
15) A mass *M* (which you may assume is a point mass) is attached to the end of a rod (length *L*) of uniform mass density and total mass *M*. The system is suspended from the ceiling with a pivot.

a) Determine the tension in the rod at the pivot and at the point P when the system is stationary.

b) Calculate the period of oscillation for small displacements from equilibrium and determine the period if the rod had a length of L = 2 m.



16) Consider two disks, with a smaller disk attached to a larger disk comprising a physical pendulum, as shown:



The large disk has a mass *M* and radius *R*, while the small disk has mass *m* and radius *r*. The center of the small disk is located on the perimeter of the large disk, and the large disk is supported at its center on a frictionless axle (coming out of the page). The system is rotated through an angle θ and released.

a) Find $v(R,g,\theta,M,m,R,r)$.

b) Find $T(R,g,M,m,R,r)$.