AP PHYSICS C: MECHANICS

UNIT 1

Kinematics

14–20%
AP EXAM WEIGHTING

~11/~22
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

Personal Progress Check 1

Multiple-Choice: ~15 questions
Free-Response: 1 question

UNIT 1

14–20% AP EXAM WEIGHTING | **~11/~22** CLASS PERIODS

Kinematics

BIG IDEA 1
Changes **CHA**

- When descending a hill on your bike, why do you roll faster the farther you go?
- Why should you throw a stone higher if you want it to go farther?

Developing Understanding

Although motion is considered an accepted phenomenon because it can easily be seen, discerning—and eventually understanding—*why* objects move requires more observation. Unit 1 introduces students to kinematics—particularly one-dimensional, two-dimensional, and projectile motion. Students will not only learn how to define each kinematic quantity (position, velocity, acceleration, and time), but also how to distinguish between them, and how to graphically and mathematically represent the relationships among them. Kinematics serves as a foundation for various physics principles and concepts, and in the units that follow, students are expected to call upon their knowledge of kinematic quantities to describe components of motion in a variety of scenarios, such as how acceleration is addressed with Newton's third law of motion.

Building the Science Practices

1.C 3.A 4.A

The ability to describe and explain physical processes, principles, and concepts is central to the study of physics. Physicists often create and use models and representations to analyze phenomena, make predictions, and communicate ideas. In this unit, students will practice demonstrating consistency between different types of representations of the same physical situation. For example, students will create a motion map, a velocity versus time graph, and a set of equations that all model the same motion of an object or a system. The content of Unit 1 provides students with multiple opportunities to discuss the relationships between variables and to model these relationships with various representations.

Unit 1 will also teach students to identify appropriate data to plot in order to describe patterns and trends in data, as patterns and trends help scientists understand relationships between fundamental laws and the world around them. Because identifying the appropriate data as well as identifying patterns gets easier with practice, introducing these skills in Unit 1 is important for student success.

Preparing for the AP Exam

Students should be able to identify, describe, and/or explain the relationships among features in graphs, free-body diagrams, and other representations. They should also be able to analyze patterns and trends in data (i.e., different motion plots) and use mathematical routines (selecting appropriate kinematic equations, deriving mathematical relationships, etc.).

Often, students studying kinematics have trouble recognizing one major difference between instantaneous and average velocity. That is, instantaneous velocity is a property of an object in motion at a single time, while the average velocity (total displacement divided by total time) of an object in motion depends on the total amount of time.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~11/~22 CLASS PERIODS
CHA-1	1.1 Kinematics: Motion in One Dimension	1.A Describe the physical meaning (includes identifying features) of a representation. 1.B Describe the relationship between different types of representations of the same physical situation. 3.A Select and plot appropriate data. 4.A Identify and describe patterns and trends in data or a graph. 4.C Linearize data and/or determine a best fit line or curve. 5.A Select an appropriate law, definition, or mathematical relationship, or model to describe a physical situation. 7.A Make a scientific claim.	
CHA-2	1.2 Kinematics: Motion in Two Dimensions	1.C Demonstrate consistency between different types of representations of the same physical situation. 5.B Determine the relationship between variables within an equation when an existing variable changes. 6.A Extract quantities from narratives or mathematical relationships to solve problems. 7.B Support a claim with evidence from experimental data.	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 1. Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 115 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	1.1	**Desktop Experiment** Provide students with a pull-back toy car and a means to take video, and have them record position versus time data for the car as it speeds up and slows down. Have students fit a cubic polynomial to the position-time data and use calculus to predict the car's maximum speed and initial and final magnitude of acceleration.
2	1.2	**Desktop Experiment** Give students a ball launcher, right-triangular block, and meterstick. Have them calculate the launch speed of the ball using a horizontal launch of the ball from the launcher, then predict where the ball will land if the ball is launched on the triangular block.
3	1.1	**Changing Representations** Give students a verbal description of segmented motion, such as "accelerates from rest at 5 m/s^2 for 10 seconds, then comes to rest again after another 20 seconds." Have students draw position/velocity/acceleration graphs and formulate piecewise position/velocity/acceleration equations of motion.
4	1.1	**Create a Plan** Find data of speed and total stopping distance for cars. Provide students with five pairs of speed and total stopping distances (not broken into thinking and braking distances). Ask students to determine from the data the driver's reaction time and the car's braking acceleration.
5	1.2	**Graph and Switch** Student A creates a horizontal and vertical pair of velocity graphs for projectile motion, and Student B must write a narrative of what happens (including whether the projectile was shot at an angle, lands higher or lower or at the same height).

Unit Planning Notes

Use the space below to plan your approach to the unit.

SUGGESTED SKILLS

Visual Representations

1.A Describe the physical meaning (includes identifying features) of a representation.

1.B Describe the relationship between different types of representations of the same physical situation.

Representing Data and Phenomena

3.A Select and plot appropriate data.

Analysis

4.A Identify and describe patterns and trends in data or a graph.

4.C Linearize data and/or determine a best fit line or curve.

Mathematical Routines

5.A Select an appropriate law, definition, or mathematical relationship, or model to describe a physical situation.

Argumentation

7.A Make a scientific claim.

AVAILABLE RESOURCES

Classroom Resources >

- AP Physics Featured Question: Projectile Concepts
- AP Physics Featured Question: Raft with Hanging Weights
- Critical Thinking Questions in Physics
- Physics Instruction Using Video Analysis Technology
- Quantitative Skills in the AP Sciences
- Teaching Strategies for Limited Class Time

TOPIC 1.1 Kinematics: Motion in One Dimension

Required Course Content

ENDURING UNDERSTANDING

CHA-1

There are relationships among the vector quantities of position, velocity, and acceleration for the motion of a particle along a straight line.

LEARNING OBJECTIVE

CHA-1.A

a. Determine the appropriate expressions for velocity and position as a function of time for an object accelerating uniformly in one dimension with given initial conditions.

b. Calculate unknown variables of motion such as acceleration, velocity, or positions for an object undergoing uniformly accelerated motion in one dimension.

c. Calculate values such as average velocity or minimum or maximum velocity for an object in uniform acceleration.

ESSENTIAL KNOWLEDGE

CHA-1.A.1

The kinematic relationships for an object accelerating uniformly in one dimension are:

$$x = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2$$

$$v_x = v_{x_0} + a_x t$$

$$v_x^2 = v_{x_0}^2 + 2a_x (x - x_0)$$

a. The constant velocity model can be derived from the above relationships.

$$v_x = \frac{\Delta x}{\Delta t}$$

b. The average velocity and acceleration models can also be derived from the above relationships.

$$v_{x(avg)} = \frac{\Delta x}{\Delta t}$$

$$a_{x(avg)} = \frac{\Delta v_x}{\Delta t}$$

continued on next page

LEARNING OBJECTIVE

CHA-1.B

Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.

ESSENTIAL KNOWLEDGE

CHA-1.B.1

Differentiation and integration are necessary for determining functions that relate position, velocity, and acceleration for an object with nonuniform acceleration.

$$v_x = \frac{dx}{dt}$$

$$a_x = \frac{dv_x}{dt}$$

a. These functions may include trigonometric, power, or exponential functions of time.

b. They may also include a velocity-dependent acceleration function (such as a resistive force).

CHA-1.C

Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.

CHA-1.C.1

Position, velocity, and acceleration versus time for a moving object are related to each other and depend on an understanding of slope, intercepts, asymptotes, and area or upon conceptual calculus concepts.

a. These functions may include trigonometric, power, exponential functions (of time) or velocity-dependent functions.

SUGGESTED SKILLS

Visual Representations

1.C Demonstrate consistency between different types of representations of the same physical situation.

Theoretical Relationships

5.B Determine the relationship between variables within an equation when an existing variable changes.

Mathematical Routines

6.A Extract quantities from narratives or mathematical relationships to solve problems.

Argumentation

7.B Support a claim with evidence from experimental data.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 1.2
Kinematics: Motion in Two Dimensions

Required Course Content

ENDURING UNDERSTANDING

CHA-2

There are multiple simultaneous relationships among the quantities of position, velocity, and acceleration for the motion of a particle moving in more than one dimension with or without net forces.

LEARNING OBJECTIVE

CHA-2.A

a. Calculate the components of a velocity, position, or acceleration vector in two dimensions.

b. Calculate a net displacement of an object moving in two dimensions.

c. Calculate a net change in velocity of an object moving in two dimensions.

d. Calculate an average acceleration vector for an object moving in two dimensions.

e. Calculate a velocity vector for an object moving relative to another object (or frame of reference) that moves with a uniform velocity.

f. Describe the velocity vector for one object relative to a second object with respect to its frame of reference.

ESSENTIAL KNOWLEDGE

CHA-2.A.1

All of the kinematic quantities are vector quantities and can be resolved into components (on a given coordinate system).

a. Vector addition and subtraction are necessary to properly determine changes in quantities.

b. The position, average velocity, and average acceleration can be represented in the following vector notation:

$$\vec{r} = \vec{x} + \vec{y} + \vec{z}$$

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

continued on next page

LEARNING OBJECTIVE

CHA-2.B

Derive an expression for the vector position, velocity, or acceleration of a particle, at some point in its trajectory, using a vector expression or using two simultaneous equations.

ESSENTIAL KNOWLEDGE

CHA-2.B.1

Differentiation and integration are necessary for determining functions that relate position, velocity, and acceleration for an object in each dimension.

$$v_x = \frac{dx}{dt}$$

$$a_x = \frac{dv_x}{dt}$$

a. The accelerations may be different in each direction and may be nonuniform.

b. The resultant vector of a given quantity such as position, velocity, or acceleration is the vector sum of the components of each quantity.

CHA-2.C

Calculate kinematic quantities of an object in projectile motion, such as displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.

CHA-2.C.1

Motion in two dimensions can be analyzed using the kinematic equations if the motion is separated into vertical and horizontal components.

a. Projectile motion assumes negligible air resistance and therefore constant horizontal velocity and constant vertical acceleration (earth's gravitational acceleration).

b. These kinematic relationships only apply to constant (uniform) acceleration situations and can be applied in both *x* and *y* directions.

$$x = x_0 + v_{x_0}t + \frac{1}{2}a_x t^2$$

$$v_x = v_{x_0} + a_x t$$

$$v_x^2 = v_{x_0}^2 + 2a_x\left(x - x_0\right)$$

CHA-2.D

Describe the motion of an object in two-dimensional motion in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.

CHA-2.D.1

The position, velocity, and acceleration versus time for a moving object are related to each other and depend on understanding of slope, intercepts, asymptotes, and area or upon conceptual calculus concepts.

AP PHYSICS C: MECHANICS

UNIT 2

Newton's Laws of Motion

17–23%
AP EXAM WEIGHTING

~12/~24
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

Personal Progress Check 2

Multiple-Choice: ~25 questions
Free-Response: 1 question

UNIT 2

17–23% AP EXAM WEIGHTING | ~12/~24 CLASS PERIODS

Newton's Laws of Motion

BIG IDEA 2
Force Interactions **INT**

- Why does the swirling motion continue after you've stopped stirring a cup of coffee or tea?
- If you apply the same amount of "push" to a car as you would a shopping cart, why doesn't it move?
- Why will the sun set tomorrow in nearly the same place that it set today?
- Why must you push backward to make a skateboard move forward?

Developing Understanding

To understand how and why objects move, students must first understand the role forces play in motion. Unit 2 investigates Newton's laws of motion, which describe the relationship among moving objects and the forces acting on them. Students will learn how forces can change the motion of an object (first law); about the relationship between force, mass, and motion (second law); and why balanced forces become unbalanced (third law). These laws form the foundation of classical mechanics, and in subsequent units, students will evolve their understanding by applying Newton's laws of motion to a variety of physics principles, including the conservation of energy (Unit 3), rotation (Unit 5), simple harmonic motion (Unit 6), and the orbital motion of satellites (Unit 7).

Building the Science Practices

1.B 2.D 7.C

The ability to create, describe, and use representations is central to the study of physics. Physicists create and use models and representations to analyze phenomena, make predictions, and communicate ideas. In Unit 2, students will describe and discuss the relationships between different types of representations (for example, mathematical models and free-body diagrams) of the same physical situation and will re-express one type of representation as another.

The laboratory focus in Unit 2 encourages students to make observations and/or collect data from representations of laboratory setups or results. Students are required to determine an appropriate experimental procedure, including sketches of the laboratory setup to answer a scientific question.

Unit 2 also provides opportunities to develop the skill of crafting scientific arguments. Scientific arguments may specify a cause-and-effect relationship between variables or describe a mechanism through which a phenomenon occurs. By the end of the unit, students should be able to identify the evidence necessary to defend a claim, be able to identify a fundamental principle of physics to begin the reasoning process, and begin to sketch out the support for a claim citing evidence from physical representations.

Preparing for the AP Exam

Students often struggle with knowing where to begin with laboratory design free-response questions. For help, we recommend they perform scaffolded activities and labs to determine the appropriate data needed to answer a scientific question. Teachers can refer to the learning objectives aligned to this unit to create these activities.

On the AP Exam, a free-response question may require students to create, use, and analyze graphs and representations (i.e., free-body force diagrams) when applying Newton's second law of motion. Students must select an appropriate law, definition, or model to describe a physical situation and/or develop a logical and coherent argument (or aspects of one). For example, when given a scenario in which two almost identical masses are hung on opposite sides of a pulley, students should be able to predict and justify why the acceleration of the system will be much smaller than the acceleration due to gravity.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~12/~24 CLASS PERIODS
INT-1	**2.1 Newton's Laws of Motion: First and Second Law**	1.A Describe the physical meaning (includes identifying features) of a representation. 2.D Make observations or collect data from representations of laboratory setups or results. 3.B Represent features of a model or the behavior of a physical system using appropriate graphing techniques, appropriate scale, and units. 4.B Demonstrate consistency between different graphical representations of the same physical situation. 5.A Select an appropriate law, definition, mathematical relationship, or model to describe a physical situation. 7.A Make a scientific claim. 7.B Support a claim with evidence from experimental data.	
INT-2	**2.2 Circular Motion**	1.B Describe the relationship between different types of representations of the same physical situation. 5.C Determine the relationship between variables within an equation when a new variable is introduced. 6.B Apply an appropriate law, definition, or mathematical relationship to solve a problem.	
INT-3	**2.3 Newton's Laws of Motion: Third Law**	1.C Demonstrate consistency between different types of representations of the same physical situation. 5.C Determine the relationship between variables within an equation when a new variable is introduced. 5.D Determine or estimate the change in a quantity using a mathematical relationship. 7.C Support a claim with evidence from physical representations. 7.D Provide reasoning to justify a claim using physical principles or laws.	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 2. Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 115 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	2.2	**Desktop Experiment** Drill a small hole in the center of a wooden meterstick so that a pencil point fits in the hole. Place a penny on the meterstick and gently rotate the meterstick faster and faster until the penny slips. Have students make measurements and calculations to find the coefficient of static friction between the meterstick and the penny.
2	2.1	**Graph and Switch** Student A produces a free-body diagram. Student B is to suggest a situation where the forces on an object would be described by that diagram.
3	2.3	**Discussion Groups** Have students explain why a strong man will win against a small child in tug-of-war, even though the rope always has the same tension at both ends. Have students support their reasoning with free-body diagrams.
4	2.1	**Desktop Experiment** Ask students to find the coefficient of friction (static or kinetic) of a shoe or other object. This activity can be made into a competition, where the team with the simplest procedure or the team that uses the least equipment wins.
5	2.1	**Desktop Experiment** Give students an object having unknown-mass (or have students use their set of house keys, if available), known masses, string, pulley, meterstick, and stopwatch. Have students determine the unknown mass of the object.

Unit Planning Notes

Use the space below to plan your approach to the unit.

SUGGESTED SKILLS

Visual Representations

1.A Describe the physical meaning (includes identifying features) of a representation.

Question and Method

2.D Make observations or collect data from representations of laboratory setups or results.

Representing Data and Phenomena

3.B Represent features of a model or the behavior of a physical system using appropriate graphing techniques, appropriate scale, and units.

Data Analysis

4.B Demonstrate consistency between different graphical representations of the same physical situation.

Theoretical Relationships

5.A Select an appropriate law, definition, mathematical relationship, or model to describe a physical situation.

Argumentation

7.A Make a scientific claim.

7.B Support a claim with evidence from experimental data.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**

continued on next page

TOPIC 2.1

Newton's Laws of Motion: First and Second Law

Required Course Content

ENDURING UNDERSTANDING

INT-1

A net force will change the translational motion of an object.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-1.A Describe an object (either in a state of equilibrium or acceleration) in different types of physical situations such as inclines, falling through air resistance, Atwood machines, or circular tracks).	**INT-1.A.1** Newton's second law can be applied to an object in accelerated motion or in a state of equilibrium.
INT-1.B a. Explain Newton's first law in qualitative terms and apply the law to many different physical situations. b. Calculate a force of unknown magnitude acting on an object in equilibrium.	**INT-1.B.1** Newton's first law is the special case of the second law. When the acceleration of an object is zero (i.e., velocity is constant or equal to zero), the object is in a state of equilibrium and the following statements are true: $\sum F_x = 0$ $\sum F_y = 0$ a. Forces can be resolved into components and these components can be separately added in their respective directions.

continued on next page

LEARNING OBJECTIVE

INT-1.C

a. Calculate the acceleration of an object moving in one dimension when a single constant force (or a net constant force) acts on the object during a known interval of time.

b. Calculate the average force acting on an object moving in a plane with a velocity vector that is changing over a specified time interval.

c. Describe the trajectory of a moving object that experiences a constant force in a direction perpendicular to its initial velocity vector.

d. Derive an expression for the net force on an object in translational motion.

e. Derive a complete Newton's second law statement (in the appropriate direction) for an object in various physical dynamic situations (e.g., mass on incline, mass in elevator, strings/pulleys, or Atwood machines).

ESSENTIAL KNOWLEDGE

INT-1.C.1

The appropriate use of Newton's second law is one of the fundamental skills in mechanics.

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

a. The second law is a vector relationship. It may be necessary to draw complete free-body diagrams to determine unknown forces acting on an object.

b. Forces acting parallel to the velocity vector have the capacity to change the speed of the object.

c. Forces acting in the perpendicular direction have the capacity to change the direction of the velocity vector.

LEARNING OBJECTIVE

INT-1.D

Calculate a value for an unknown force acting on an object accelerating in a dynamic situation (e.g., inclines, Atwood machines, falling with air resistance, pulley systems, mass in elevator, etc.).

ESSENTIAL KNOWLEDGE

INT-1.D.1

Using appropriate relationships derived from a Newton's second law analysis, unknown forces (or accelerations) can be determined from the given known physical characteristics.

continued on next page

AVAILABLE RESOURCES

Classroom Resources >

- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-1.E a. Describe the relationship between frictional force and the normal force for static friction and for kinetic friction. b. Explain when to use the static frictional relationship versus the kinetic frictional relationship in different physical situations (e.g., object sliding on surface or object not slipping on incline).	**INT-1.E.1** The relationship for the frictional force acting on an object on a rough surface is: $\left\|\vec{F}_{f_s}\right\| \le \mu_s \left\|\vec{F}_N\right\|$ $\left\|\vec{F}_{f_k}\right\| = \mu_k \left\|\vec{F}_N\right\|$
INT-1.F Describe the direction of frictional forces (static or kinetic) acting on an object under various physical situations.	**INT-1.F.1** The direction of friction can be determined by the relative motion between surfaces in kinetic frictional cases. a. In cases where the direction of friction is not obvious or is not directly evident from relative motion, then the net motion of the object and the other forces acting on the object are required to determine the direction of the frictional force.
INT-1.G a. Derive expressions that relate mass, forces, or angles of inclines for various slipping conditions with friction. b. Calculate the value for the static frictional force for an object in various dynamic situations (e.g., an object at rest on truck bed, an object at rest on incline, or an object pinned to a horizontal surface).	**INT-1.G.1** The maximum value of static friction has a precise relationship: $\left\|\vec{F}_{f_s}\right\| = \mu_s \left\|\vec{F}_N\right\|$ a. This relationship can be used to determine values such as, "The maximum angle of incline at which the block will not slip."

continued on next page

LEARNING OBJECTIVE

INT-1.H

a. Derive an expression for the motion of an object freely falling with a resistive drag force (or moving horizontally subject to a resistive horizontal force).

b. Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).

ESSENTIAL KNOWLEDGE

INT-1.H.1

The standard "resistive force" in this course is defined as a velocity-dependent force in the opposite direction of velocity, for example:

$$\vec{F}_r = -k\vec{v}$$

or

$$\left|\vec{F}_r\right| = kv^2$$

INT-1.I

Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.

INT-1.I.1

The terminal velocity is defined as the maximum speed achieved by an object falling under the influence of a given drag force. The terminal condition is reached when the magnitude of the drag force is equal to the magnitude of the weight of the object.

INT-1.J

a. Derive a differential equation for an object in motion subject to a specified resistive force.

b. Derive an expression for a time-dependent velocity function for an object moving under the influence of a given resistive force (with given initial conditions).

c. Derive expressions for the acceleration or position of an object moving under the influence of a given resistive force.

INT-1.J.1

Because the resistive force is a function of velocity, applying Newton's second law correctly will lead to a differential equation for velocity. This is an example of that statement:

$$\frac{dv}{dt} = -\frac{k}{m}v$$

a. Using the method of separation of variables, the velocity can be determined from relationships by correctly integrating over the proper limits of integration.

b. The acceleration or position can be determined using methods of calculus once a function for velocity is determined.

SUGGESTED SKILLS

Visual Representations

1.B Describe the relationship between different types of representations of the same physical situation.

Theoretical Relationships

5.C Determine the relationship between variables within an equation when a new variable is introduced.

Mathematical Routines

6.B Apply an appropriate law, definition, or mathematical relationship to solve a problem.

AVAILABLE RESOURCES

Classroom Resources >

- AP Physics Featured Question: Projectile Concepts
- AP Physics Featured Question: Raft with Hanging Weights
- Critical Thinking Questions in Physics
- Physics Instruction Using Video Analysis Technology
- Quantitative Skills in the AP Sciences
- Teaching Strategies for Limited Class Time

TOPIC 2.2

Circular Motion

Required Course Content

ENDURING UNDERSTANDING

INT-2

The motion of some objects is constrained so that forces acting on the object cause it to move in a circular path.

LEARNING OBJECTIVE

INT-2.A

a. Calculate the velocity of an object moving in a horizontal circle with a constant speed, when subject to a known centripetal force.

b. Calculate relationships among the radius of a circle, the speed of an object (or period of revolution), and the magnitude of centripetal acceleration for an object moving in uniform circular motion.

ESSENTIAL KNOWLEDGE

INT-2.A.1

Centripetal acceleration is defined by:

$$a_c = \frac{v^2}{r}$$

or defined using angular velocity:

$$a_c = \omega^2 r$$

a. Uniform circular motion is defined as an object moving in a circle with a constant speed.

b. The net force acting in the radial direction can be determined by applying Newton's second law in the radial direction.

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-2.B a. Explain how a net force in the centripetal direction can be a single force, more than one force, or even components of forces that are acting on an object moving in circular motion. b. Describe forces that are exerted on objects undergoing horizontal circular motion, vertical circular motion, or horizontal circular motion on a banked curve. c. Describe forces that are acting on different objects traveling in different circular paths.	**INT-2.B.1** In order for an object to undergo circular motion in any context, there must be a force, multiple forces, or components of forces acting in the radial direction. These forces can be represented with appropriate free-body diagrams.
INT-2.C a. Describe the direction of the velocity and acceleration vector for an object moving in two dimensions, circular motion, or uniform circular motion. b. Calculate the resultant acceleration for an object that changes its speed as it moves in a circular path.	**INT-2.C.1** An object that changes directions will always have an acceleration component that is perpendicular to the velocity vector. The velocity vector will always be tangential to the path of the particle. a. As an object moves in a circle with changing speed, the resultant acceleration, at any point, is the vector sum of the radial acceleration and tangential acceleration.
INT-2.D Derive expressions relating centripetal force to the minimum speed or maximum speed of an object moving in a vertical circular path.	**INT-2.D.1** The centripetal force is provided only by the gravitational force for an object moving at minimum speed at the top of a vertical circle. This speed is called "critical speed" in certain textbooks. a. The maximum speed occurs at the bottom of the circle and is related to all of the vertical forces acting on the object.
INT-2.E Derive expressions relating the centripetal force to the maximum speed of an object or minimum speed of an object moving in a circular path on a banked surface with friction.	**INT-2.E.1** Components of the static friction force and the normal force can contribute to the centripetal force for an object traveling in a circle on a banked surface.

SUGGESTED SKILLS

Visual Representations

1.C Demonstrate consistency between different types of representations of the same physical situation.

Theoretical Relationships

5.C Determine the relationship between variables within an equation when a new variable is introduced.

5.D Determine or estimate the change in a quantity using a mathematical relationship.

Argumentation

7.C Support a claim with evidence from physical representations.

7.D Provide reasoning to justify a claim using physical principles or laws.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 2.3

Newton's Laws of Motion: Third Law

Required Course Content

ENDURING UNDERSTANDING

INT-3

There are force pairs with equal magnitude and opposite directions between any two interacting objects.

LEARNING OBJECTIVE

INT-3.A

a. Describe the forces of interaction between two objects (Newton's third law).

b. Describe pairs of forces that occur in a physical system due to Newton's third law.

c. Describe the forces that occur between two (or more) objects accelerating together (e.g., in contact or connected by light strings, springs, or cords).

ESSENTIAL KNOWLEDGE

INT-3.A.1

The forces exerted between objects are equal in magnitude and opposite in direction.

a. Third law force pairs are always internal to the system of the two objects that are interacting.

b. Each force in the pair is always the same type of force.

LEARNING OBJECTIVE

INT-3.B

Derive expressions that relate the acceleration of multiple connected masses moving in a system (e.g., Atwood machines) connected by light strings with tensions (and pulleys).

ESSENTIAL KNOWLEDGE

INT-3.B.1

To analyze a complete system of multiple connected masses in motion, several applications of Newton's second law in conjunction with Newton's third law may be necessary. This may involve solving two or three simultaneous linear equations.

AP PHYSICS C: MECHANICS

UNIT 3

Work, Energy, and Power

14–17%
AP EXAM WEIGHTING

~10/~20
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

Personal Progress Check 3

Multiple-Choice: ~20 questions
Free-Response: 1 question

UNIT 3

14–17% AP EXAM WEIGHTING | ~10/~20 CLASS PERIODS

Work, Energy, and Power

BIG IDEA 2

Force Interactions **INT**

- Why is no work done when you push against a wall, but work is done when you coast down a hill?

BIG IDEA 4

Conservation **CON**

- Why does a stretched rubber band return to its original length?
- Why is it easier to walk up a flight of steps, rather than run, when the gravitational potential energy of the system is the same?

Developing Understanding

Are you working hard, or hardly working? The answer depends on how you define *work*. In Unit 3, students will explore the relationship between work, energy, and power and will be introduced to the principle of conservation as a foundational model of physics, as well as the concept of work as an agent of change for energy. Students are not only expected to functionally define and calculate work, energy, and power, but must also be comfortable graphically and mathematically representing them. Understanding these relationships will help students make connections to other content presented in the course. For instance, students can use the concept of work to link the principles of energy transfer, forces, momentum, and certain kinematic equations.

Building the Science Practices

2.E 5.C 6.C

Scientific questions can range in scope as well as specificity. Students will identify or describe the appropriate principle needed to answer a specific scientific question, as well as the potential sources of experimental uncertainty. Students will identify and/or describe potential sources of experimental error either in their own experiments or when given an experimental setup.

Students will develop the practice of argumentation by predicting the causes or effects of a change in, or disruption to, one or more components in a system. It is essential that students are able to clearly demonstrate the ability to determine the relationship between variables within an equation when a new variable is introduced. Students also need scaffolded practice with clearly calculating unknown quantities from known quantities from selecting and following logical computational pathways. Students who are unable to be clear in their calculations and derivations will lose points on questions that ask them to calculate—or in other words, to clearly show their computational steps.

Preparing for the AP Exam

When answering free-response questions, students should be able to select relevant features of a graph to describe a physical situation (e.g., graphs of force versus position, where the area under the curve is equal to the work done). Students should also be able to derive symbolic expressions from known quantities, as well as apply the appropriate laws, definitions, and/or mathematical relationships to perform a calculation or solve a problem. For example, students could be asked to apply the work-energy theorem to a scenario or calculate the negative integral of force with respect to position to determine the change in potential energy of a system.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~10/~20 CLASS PERIODS
INT-4	3.1 Work-Energy Theorem	2.A Identify a testable scientific question or problem. 7.C Support a claim with evidence from physical representations.	
CON-1	3.2 Forces and Potential Energy	1.D Select relevant features of a representation to answer a question or solve a problem. 4.B Demonstrate consistency between different graphical representations of the same physical situation. 6.A Extract quantities from narratives or mathematical relationships to solve problems.	
CON-2	3.3 Conservation of Energy	2.E Identify or describe potential sources of experimental error. 4.D Select relevant features of a graph to describe a physical situation or solve problems. 5.C Determine the relationship between variables within an equation when a new variable is introduced. 6.C Calculate an unknown quantity with units from known quantities by selecting and following a logical computational pathway. 7.E Explain the connection between experimental results and larger physical principles, laws, or theories.	
CON-3	3.4 Power	5.D Determine or estimate the change in a quantity using a mathematical relationship.	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 3. Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 115 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	3.3	**Bar Chart** Present students with a sequence of two or three energy bar charts and have them describe a realistic situation that would involve those energy transformations. Students must also draw a diagram of the situation.
2	3.2	**Graph and Switch** Student A constructs a potential energy (PE) function that has at least one minimum and a graph of that function. Student B formulates AP-level questions about the PE function (i.e., "If a 2 kg mass is released at $x = 3$ m, what is its speed at $x = 9$ m?") that Student C must answer.
3	3.3	**Desktop Experiment** Using spring-loaded suction cup launchers, have students measure the spring constant of the spring, not by removing the spring from the launcher, but by measuring some aspect of the suction cup's motion after being launched.
4	3.4	**Identify Subtasks** Have students construct graph of power delivered to a car as a function of time as the car accelerates from rest and reaches full speed. Next, ask students to determine the car's mass and its velocity as a function of time.
5	3.3	**Changing Representations** Have each student describe an everyday activity that involves the transfer of mechanical energy. Students then construct energy bar charts showing the exchanges of energy and free-body diagrams to show the forces doing work, and flowcharts to show the flow of energy from one system or form to another.

Unit Planning Notes

Use the space below to plan your approach to the unit.

SUGGESTED SKILLS

Question and Method

2.A Identify a testable scientific question or problem.

Argumentation

7.C Support a claim with evidence from physical representations.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 3.1
Work-Energy Theorem

Required Course Content

ENDURING UNDERSTANDING

INT-4

When a force is exerted on an object, and the energy of the object changes, then work was done on the object.

LEARNING OBJECTIVE

INT-4.A

a. Calculate work done by a given force (constant or as a given function *F(x)*) on an object that undergoes a specified displacement.

b. Describe the work done on an object as the result of the scalar product between force and displacement.

c. Explain how the work done on an object by an applied force acting on an object can be negative or zero.

ESSENTIAL KNOWLEDGE

INT-4.A.1

The component of the displacement that is parallel to the applied force is used to calculate the work.

a. The work done on an object by a force can be calculated using:

$$W = \int_a^b \vec{F}(r) \cdot d\vec{r}$$

b. Work is a scalar value that can be positive, negative, or zero.

c. The definition of work can be applied to an object when that object can be modeled as a point-like object.

LEARNING OBJECTIVE

INT-4.B

Calculate a value for work done on an object from a force versus position graph.

ESSENTIAL KNOWLEDGE

INT-4.B.1

The area under the curve of a force versus position graph is equivalent to the work done on the object or system.

continued on next page

LEARNING OBJECTIVE

INT-4.C

a. Calculate the change in kinetic energy due to the work done on an object or a system by a single force or multiple forces.

b. Calculate the net work done on an object that undergoes a specified change in speed or change in kinetic energy.

c. Calculate changes in an object's kinetic energy or changes in speed that result from the application of specified forces.

ESSENTIAL KNOWLEDGE

INT-4.C.1

The net work done on an (point-like) object is equal to the object's change in the kinetic energy.

$$W_{net} = \Delta K$$

a. This is defined as the work-energy theorem. The work-energy theorem can be used when an object or system can be modeled as a point-like particle (i.e., nondeformable and not having the capacity for internal energy).

b. The definition of kinetic energy is:

$$K = \frac{1}{2}mv^2$$

c. Net work done on an object is equivalent to the sum of the individual work done on an object by each of the forces acting on the object (including conservative forces).

SUGGESTED SKILLS

Visual Representations

1.D Select relevant features of a representation to answer a question or solve a problem.

Data Analysis

4.B Demonstrate consistency between different graphical representations of the same physical situation.

Mathematical Routines

6.A Extract quantities from narratives or mathematical relationships to solve problems.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 3.2
Force and Potential Energy

Required Course Content

ENDURING UNDERSTANDING

CON-1

Conservative forces internal to the system can change the potential energy of that system.

LEARNING OBJECTIVE

CON-1.A

a. Compare conservative and dissipative forces.

b. Describe the role of a conservative force or a dissipative force in a dynamic system.

ESSENTIAL KNOWLEDGE

CON-1.A.1

A force can be defined as a conservative force if the work done on an object by the force depends only on the initial and final position of the object.

a. The work done by a conservative force will be zero if the object undergoes a displacement that completes a complete closed path.

b. Common dissipative forces discussed in this course are friction, resistive forces, or externally applied forces from some object external to the system.

continued on next page

LEARNING OBJECTIVE

CON-1.B

a. Explain how the general relationship between potential energy functions and conservative forces is used to determine relationships between the two physical quantities.

b. Derive an expression that represents the relationship between a conservative force acting in a system on an object to the potential energy of the system using the methods of calculus.

ESSENTIAL KNOWLEDGE

CON-1.B.1

A definition that relates conservative forces internal to the system to the potential energy function of the system is:

$$\Delta U = -\int_a^b \vec{F}_{cf} \cdot d\vec{r}$$

a. The differential version (in one dimension) of this relationship is:

$$F_x = -\frac{dU(x)}{dx}$$

CON-1.C

Describe the force within a system and the potential energy of a system.

CON-1.C.1

The general relationship between a conservative force and a potential energy function can be described qualitatively and graphically. For example, basic curve sketching principles can be applied to generate a sketch (e.g., slopes, area under the curve, intercepts, etc.).

CON-1.D

a. Derive the expression for the potential energy function of an ideal spring.

b. Derive an expression for the potential energy function of a nonideal spring that has a nonlinear relationship with position.

CON-1.D.1

An ideal spring acting on an object is an example of a conservative force within a system (spring-object system). The ideal spring relationship is modeled by the following law and is also called “linear spring:”

$$\vec{F}_s = -k\Delta\vec{x}$$

a. Using the general relationship between conservative force and potential energy, the potential energy for an ideal spring can be shown as:

$$U_s = \frac{1}{2}k(\Delta x)^2$$

b. Nonlinear spring relationships can also be explored. These nonlinear forces are conservative since they are internal to the system (of spring-object) and dependent on position.

continued on next page

LEARNING OBJECTIVE

CON-1.E

Calculate the potential energy of a system consisting of an object in a uniform gravitational field.

ESSENTIAL KNOWLEDGE

CON-1.E.1

The definition of the gravitational potential energy of a system consisting of the Earth and on object of mass *m* near the surface of the Earth is:

$$\Delta U_g = mg\Delta h$$

LEARNING OBJECTIVE

CON-1.F

Derive an expression for the gravitational potential energy of a system consisting of a satellite or large mass (e.g., an asteroid) and the Earth at a great distance from the Earth.

ESSENTIAL KNOWLEDGE

CON-1.F.1

Using the relationship between the conservative force and potential energy, it can be shown that the gravitational potential energy of the object-Earth system is:

$$U_G = -\frac{Gm_1m_2}{r}$$

a. The potential energy of the Earth-mass system is defined to be zero at an infinite distance from the Earth.

TOPIC 3.3

Conservation of Energy

Required Course Content

ENDURING UNDERSTANDING

CON-2

The energy of a system can transform from one form to another without changing the total amount of energy in the system.

LEARNING OBJECTIVE

CON-2.A

a. Describe physical situations in which mechanical energy of an object in a system is converted to other forms of energy in the system.

b. Describe physical situations in which the total mechanical energy of an object in a system changes or remains constant.

ESSENTIAL KNOWLEDGE

CON-2.A.1

If only forces internal to the system are acting on an object in a physical system, then the total change in mechanical energy is zero.

a. Total mechanical energy is defined as the sum of potential and kinetic energy:

$$E = U_g + K + U_s$$

b. When nonconservative forces are acting on the system, the work they do changes the total energy of the system as follows:

$$W_{nc} = \Delta E$$

LEARNING OBJECTIVE

CON-2.B

Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a "conservative" mechanical system.

ESSENTIAL KNOWLEDGE

CON-2.B.1

In systems in which no external work is done, the total energy in that system is a constant. This is sometimes called a "conservative system."

a. Some common systems that are frequently analyzed in this way are systems such as pendulum systems, ball/rollercoaster track, frictionless ramps or tracks, or the mass/spring oscillator.

continued on next page

SUGGESTED SKILLS

Question and Method

2.E Identify or describe potential sources of experimental error.

Data Analysis

4.D Select relevant features of a graph to describe a physical situation or solve problems.

Theoretical Relationships

5.C Determine the relationship between variables within an equation when a new variable is introduced.

Mathematical Routines

6.C Calculate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.

Argumentation

7.E Explain the connection between experimental results and larger physical principles, laws, or theories.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
CON-2.C a. Calculate unknown quantities (e.g., speed or positions of an object) that are in a conservative system of connected objects, such as the masses in an Atwood machine, masses connected with pulley/string combinations, or the masses in a modified Atwood machine. b. Calculate unknown quantities, such as speed or positions of an object that is under the influence of an ideal spring. c. Calculate unknown quantities, such as speed or positions of an object that is moving under the influence of some other nonconstant one-dimensional force.	**CON-2.C.1** The application of the conservation of total mechanical energy can be used in many physical situations.
CON-2.D Derive expressions such as positions, heights, angles, and speeds for an object in vertical circular motion or pendulum motion in an arc.	**CON-2.D.1** In some cases, both Newton's second law and conservation of energy must be applied simultaneously to determine unknown physical characteristics in a system. One such example frequently explored is an object in a vertical circular motion in the Earth's gravity. A full treatment of force analysis and energy analysis would be required to determine some of the unknown features of the motion, such as the speed of the object at certain locations in the circular path.

TOPIC 3.4
Power

Required Course Content

ENDURING UNDERSTANDING

CON-3

The energy of an object or a system can be changed at different rates.

LEARNING OBJECTIVE

CON-3.A

a. Derive an expression for the rate at which a force does work on an object.

b. Calculate the amount of power required for an object to maintain a constant acceleration.

c. Calculate the amount of power required for an object to be raised vertically at a constant rate.

ESSENTIAL KNOWLEDGE

CON-3.A.1

Power is defined by the following expressions:

a. $P = \frac{dE}{dt}$

b. $P = \vec{F} \cdot \vec{v}$

SUGGESTED SKILLS

Theoretical Relationships

5.D Determine or estimate the change in a quantity using a mathematical relationship.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

AP PHYSICS C: MECHANICS

UNIT 4

Systems of Particles and Linear Momentum

14–17%
AP EXAM WEIGHTING

~10/~20
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

Personal Progress Check 4

Multiple-Choice: ~15 questions
Free-Response: 1 question

UNIT 4

14–17% AP EXAM WEIGHTING

~10/~20 CLASS PERIODS

Systems of Particles and Linear Momentum

BIG IDEA 1

Changes **CHA**

- Why do pictures hung on a wall sometimes tilt forward?
- Why will you fall if you lean too far over a bannister or ledge?

BIG IDEA 2

Force Interactions **INT**

- Why does water move a ship forward when its propellers push water backward?

BIG IDEA 4

Conservation **CON**

- Why are cannon barrels so much longer and heavier than cannonballs?

Developing Understanding

Have you ever wondered how a tennis player times a return shot? Alongside skill, players must consider a number of factors to estimate how far, fast, or high their swings should be. Unit 4 introduces students to these factors through the concepts of center of mass, impulse and momentum, and the conservation of linear momentum. Students will learn the relationship between impulse and momentum via application or calculations. The conservation of linear momentum and how it's applied to collisions is also addressed. Unit 4 offers a complete picture of the motion of a system, which is explored primarily through impulse and changes in momentum. Students will further their understanding of momentum and angular momentum in Unit 7 as they begin to articulate orbital and rotational motion.

Building the Science Practices

1.E 2.C 7.D

The analysis, interpretation, and application of quantitative information are vital skills for students in AP Physics. Opportunities for scientific inquiry should be designed and implemented with increasing student involvement to help enhance the development of inquiry learning, critical thinking, and problem-solving skills. The laboratory focus in Unit 4 encourages students to identify appropriate experimental procedures to answer scientific questions. Students are required to clearly and concisely determine an appropriate experimental procedure, including sketches of the laboratory setup.

Unit 4 continues to challenge students to work with representations to help describe what happens when the conditions of a scenario are changed. It is important students have scaffolded practice with modifying assumptions or conditions and making predictions about the results. It is also essential that students can use fundamental principles of physics as evidence to defend and justify their claims.

Preparing for the AP Exam

To earn full credit on a free response question, students must demonstrate competency of the learning objectives. This can be achieved by having ample practice and opportunities to apply the science practices. For instance, students should be able to articulate assumptions and limits of a representation of a physical situation, including assuming symmetry of objects and systems and describing the limitations of conservation of kinetic energy in certain collisions. They should also be able to derive a symbolic expression from known quantities. For example, students should be able to derive a symbolic expression for the final velocity of two boxes after a collision.

Students should also be given multiple opportunities to develop a coherent and logical argument (or aspects of one) and to use physical principles and/or empirical data to justify a claim or prediction. An example of this practice includes verifying the law of conservation of momentum during an explosion.

UNIT AT A GLANCE

Enduring Understandings	Topic	Suggested Skills	Class Periods ~10/~20 CLASS PERIODS
CHA-3	**4.1** Center of Mass	**6.B** Apply an appropriate law, definition, or mathematical relationship to solve a problem.	
INT-5	**4.2** Impulse and Momentum	**1.C** Demonstrate consistency between different types of representations of the same physical situation. **2.C** Identify appropriate experimental procedures to test a claim or prediction (which may include a sketch of a lab setup). **5.D** Determine or estimate the change in a quantity using a mathematical relationship.	
CON-4	**4.3** Conservation of Linear Momentum, Collisions	**1.E** Describe the effects of modifying conditions or features of a representation of a physical situation. **5.E** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. **7.D** Provide reasoning to justify a claim using physical principles or laws. **7.E** Explain the connection between experimental results and larger physical principles, laws, or theories. **7.F** Explain how potential sources of experimental error may affect results and/or conclusions.	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 4. Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 115 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	4.2	**Four-Square Problem Solving** Present students with some problem where an object's motion changes (such as a car on an on-ramp entering a freeway). Have students determine the force applied to the object using Newton's laws of motion, work-energy theorem, and impulse-momentum theorem. The fourth square is a free-body diagram.
2	4.3	**Desktop Experiment** Give students two spring-loaded carts with different mass First, have students determine the amount of kinetic energy gained by Cart 1 when launched by its spring. Then, have students make Cart 1 collide elastically with Cart 2 and predict where Cart 2 will land when it rolls off of the track.
3	4.1	**Desktop Experiment** Using two bathroom scales and a long wooden plank, have students determine the location of their center of mass. Have students determine how far their center of mass moves as they move their arms from their sides to up over their head.
4	4.3	**Desktop Experiment** Give students a device that fires a projectile much faster than can be measured directly using distance and time data. Students are to fire the projectile into a stationary, freely movable object; make necessary measurements; and use conservation of momentum to determine the launch speed of the projectile.

Unit Planning Notes

Use the space below to plan your approach to the unit.

SUGGESTED SKILLS

Mathematical Routines

6.B Apply an appropriate law, definition, or mathematical relationship to solve a problem.

AVAILABLE RESOURCES

Classroom Resources >

- AP Physics Featured Question: Projectile Concepts
- AP Physics Featured Question: Raft with Hanging Weights
- Critical Thinking Questions in Physics
- Physics Instruction Using Video Analysis Technology
- Quantitative Skills in the AP Sciences
- Teaching Strategies for Limited Class Time

TOPIC 4.1
Center of Mass

Required Course Content

ENDURING UNDERSTANDING

CHA-3

The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass.

LEARNING OBJECTIVE

CHA-3.A

a. Calculate the center of mass of a system of point masses or a system of regular symmetrical objects.

b. Calculate the center of mass of a thin rod of nonuniform density using integration.

ESSENTIAL KNOWLEDGE

CHA-3.A.1

A symmetrical, regular solid of uniform mass density has a center of mass at its geometric center.

a. For a nonuniform solid that can be considered as a collection of regular masses or for a system of masses:

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

b. The calculus definition is:

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
CHA-3.B Describe the motion of the center of the mass of a system for various situations.	**CHA-3.B.1** If there is no net force acting on an object or a system, the center of mass does not accelerate; therefore, the velocity of the center of mass remains unchanged. a. A system of multiple objects can be represented as one single mass with a position represented by the center of mass. b. The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass.
CHA-3.C Explain the difference between the terms "center of gravity" and "center of mass," and identify physical situations when these terms have identical positions and when they have different positions.	**CHA-3.C.1** The center of gravity is not precisely the same scientific quantity as the center of mass. If the object experiencing a gravitational interaction with a large planet is of large dimensions (comparable to the planet), then the gravitational acceleration due to the large planet will be a nonuniform value over the length of the object. This would result in the center of gravity location being a different location than the center of mass.

SUGGESTED SKILLS

Visual Representations

1.C Demonstrate consistency between different types of representations of the same physical situation.

Question and Method

2.C Identify appropriate experimental procedures to test a claim or prediction (which may include a sketch of a lab setup).

Theoretical Relationships

5.D Determine or estimate the change in a quantity using a mathematical relationship.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 4.2

Impulse and Momentum

Required Course Content

ENDURING UNDERSTANDING

INT-5

An impulse exerted on an object will change the linear momentum of the object.

LEARNING OBJECTIVE

INT-5.A

a. Calculate the total momentum of an object or a system of objects.

b. Calculate relationships between mass, velocity, and linear momentum of a moving object.

ESSENTIAL KNOWLEDGE

INT-5.A.1

For a single object moving with some velocity, momentum is defined as:

$$\vec{p} = m\vec{v}$$

a. The total momentum of the system is the vector sum of the momenta of the individual objects. The rate of change of momentum is equal to the net external force.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

LEARNING OBJECTIVE

INT-5.B

Calculate the quantities of force, time of collision, mass, and change in velocity from an expression relating impulse to change in linear momentum for a collision of two objects.

ESSENTIAL KNOWLEDGE

INT-5.B.1

Impulse is defined as the average force acting over a time interval:

$$\vec{J} = \vec{F}_{avg}\Delta t$$

a. Impulse is also equivalent to the change in momentum of the object receiving the impulse.

$$\int \vec{F}\,dt = \Delta\vec{p} = \vec{J}$$

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-5.C Describe relationships between a system of objects' individual momenta and the velocity of the center of mass of the system of objects.	**INT-5.C.1** A collection of objects with individual momenta can be described as one system with one center of mass velocity.
INT-5.D Calculate the momentum change in a collision using a force versus time graph for a collision.	**INT-5.D.1** Impulse is equivalent to the area under a force versus time graph.
INT-5.E Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.	**INT-5.E.1** Momentum changes can be calculated using the calculus relationship for impulse: $\vec{J} = \Delta\vec{p} = \int \vec{F}\, dt$

SUGGESTED SKILLS

Visual Representations

1.E Describe the effects of modifying conditions or features of a representation of a physical situation.

Theoretical Relationships

5.E Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

Argumentation

7.D Provide reasoning to justify a claim using physical principles or laws.

7.E Explain the connection between experimental results and larger physical principles, laws, or theories.

7.F Explain how potential sources of experimental error may affect results and/or conclusions.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 4.3
Conservation of Linear Momentum, Collisions

Required Course Content

ENDURING UNDERSTANDING

CON-4

In the absence of an external force, the total momentum within a system can transfer from one object to another without changing the total momentum in the system.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
CON-4.A a. Calculate the velocity of one part of a system after an explosion or a collision of the system. b. Calculate energy changes in a system that undergoes a collision or an explosion.	**CON-4.A.1** Total momentum is conserved in the system and momentum is conserved in each direction in the absence of an external force.
CON-4.B Calculate the changes of momentum and kinetic energy as a result of a collision between two objects.	**CON-4.B.1** In the absence of an external force, momentum is always conserved. a. Kinetic energy is only conserved in elastic collisions. b. In an inelastic collision, some kinetic energy is transferred to internal energy of the system.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
CON-4.C Describe the quantities that are conserved in a collision.	**CON-4.C.1** Momentum is a vector quantity. a. Momentum in each dimension is conserved in the absence of a net external force exerted on the object or system. b. Kinetic energy is conserved only if the collision is totally elastic.
CON-4.D Calculate the speed of the center of mass of a system.	**CON-4.D.1** Forces internal to a system do not change the momentum of the center of mass.
CON-4.E a. Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects in all types of collisions (elastic or inelastic) in one dimension, given the initial conditions of the objects. b. Derive expressions for the conservation of momentum for a particular collision in one dimension.	**CON-4.E.1** Conservation of momentum states that the momentum of a system remains constant when there are no external forces exerted on the system. a. Momentum is a vector quantity. b. An elastic collision is defined as a system where the total kinetic energy is conserved in the collision.
CON-4.F a. Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects involved in a two-dimensional collision (including an elastic collision), given the initial conditions of the objects. b. Derive expressions for the conservation of momentum for a particular two-dimensional collision of two objects.	**CON-4.F.1** In the absence of a net external force during an interaction, linear momentum is conserved. a. Momentum is a vector quantity. The momenta in each dimension (horizontal and vertical) are also conserved. b. Using momentum components can be useful in this approach.

AP PHYSICS C: MECHANICS

UNIT 5

Rotation

14–20%
AP EXAM WEIGHTING

~10/~20
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

Personal Progress Check 5

Multiple-Choice: ~20 questions
Free-Response: 1 question

UNIT 5

14–20% AP EXAM WEIGHTING

~10/~20 CLASS PERIODS

Rotation

BIG IDEA 2
Force Interactions **INT**

- Why does a curveball take less time to reach the plate than a fastball?
- Why is it easier to balance a bicycle when it's in motion?

BIG IDEA 4
Conservation **CON**

- How can you increase your swing on a swing set without being pushed?

Developing Understanding

In this unit, students will investigate torque and rotational statics, kinematics, and dynamics, in addition to angular momentum and its conservation, to gain an in-depth and comprehensive understanding of rotation. Students are provided with opportunities to make connections between the content and models explored in the first four units, as well as with opportunities to demonstrate the analogy between translational and rotational kinematics. Unfortunately, when dealing with rotational motion, all the conceptual difficulties found in translational motion also have direct analogs. For example, if the angular velocity is zero, students often believe that the angular acceleration must also be zero. Astronomical phenomena (such as satellites in orbit) are explored in Unit 7 to build students' knowledge of angular momentum and its conservation.

Building the Science Practices

3.C 5.D 6.D

In this unit, students will create and use graphical representations to demonstrate understanding of the functional relationships between the variables that describe the motion of objects or systems. The content of Unit 5 provides students with multiple opportunities to discuss the relationships between variables and to graph these relationships in various scenarios.

Unit 5 will also help students master clear and concise derivations of a change in quantity using mathematical relationships. On the AP Exam, students should be comfortable with calculating an unknown quantity with units, and/or a symbolic expression from known quantities, by selecting and following a logical computational pathway. Students should also be able to assess the reasonableness of their results or solutions and will be expected to derive and/or calculate on the free-response section on the AP Physics C: Mechanics Exam.

Preparing for the AP Exam

To earn full credit on a free-response question, students must be able to describe and modify their assumptions about a representation of a physical situation. They should also be able to determine the change in a quantity using a mathematical relationship and calculate a symbolic expression from known quantities. For example, students should be able to determine the change in angular momentum after torque has been applied to a pulley.

Calculating an unknown quantity with units, and/or a symbolic expression from known quantities, by selecting and following a logical computational pathway is also required. For example, students must be able to calculate the angular acceleration of a hoop rolling without sliding down an incline. Quite often when prompted to "calculate," students will fail to properly show all mathematical work. They should start with a known physics formula and show, step-by-step (including numeric substitutions), how a final answer is achieved.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~10/~20 CLASS PERIODS
INT-6	5.1 Torque and Rotational Statics	2.D Make observations or collect data from representations of laboratory setups or results. 3.B Represent features of a model or the behavior of a physical system using appropriate graphing techniques, appropriate scale, and units.	
CHA-4	5.2 Rotational Kinematics	2.B Make a claim or predict the results of an experiment. 5.B Determine the relationship between variables within an equation when an existing variable changes. 6.C Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.	
INT-7	5.3 Rotational Dynamics and Energy	1.E Describe the effects of modifying conditions or features of a representation of a physical situation. 3.C Sketch a graph that shows a functional relationship between two quantities. 4.D Select relevant features of a graph to describe a physical situation or solve problems. 5.D Determine or estimate the change in a quantity using a mathematical relationship.	
CON-5	5.4 Angular Momentum and Its Conservation	1.E Describe the effects of modifying conditions or features of a representation of a physical situation. 5.E Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. 6.D Assess the reasonableness of results or solutions. 7.D Provide reasoning to justify a claim using physical principles or laws.	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 5. Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 115 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	5.3	**Desktop Experiment** Students allow a yo-yo to fall and unroll. Have them use a meterstick and stopwatch to determine its downward acceleration. Also have them measure its mass and the radius of its axle and use that information to determine the yo-yo's rotational inertia using rotational dynamics.
2	5.3	**Desktop Experiment** Have students release a yo-yo from the top of a ramp and allow it to roll down the ramp. Have them use a meterstick and stopwatch to determine the yo-yo's final velocity and the height of its release. Next, have them measure the yo-yo's outer radius and mass and use that information to determine the yo-yo's rotational inertia using energy concepts.
3	5.4	**Create a Plan** Have students complete the necessary research to determine the rotational inertia of a human body in different configurations (arms outstretched, arms pulled in, for example). Then, obtain footage of a figure skater spinning and pulling in her his/arms. Have students analyze the footage to see if angular momentum is conserved.
4	5.3	**Bar Chart** Have students a hoop and a disk (equal mass and radius) down identical ramps. Then have them explain why the disk reached the bottom in less time using energy bar charts and to-scale free-body diagrams.
5	5.1	**Identify Subtasks** Have students design a walkway (of given mass) that is to be suspended from a ceiling. Have them determine the amount of force the two supports (one on each end) must be able to provide as a person (of given mass) walks across the walkway.

Unit Planning Notes

Use the space below to plan your approach to the unit.

SUGGESTED SKILLS

Question and Method

2.D Make observations or collect data from representations of laboratory setups or results.

Representing Data and Phenomena

3.B Represent features of a model or the behavior of a physical system using appropriate graphing techniques, appropriate scale, and units.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 5.1
Torque and Rotational Statics

Required Course Content

ENDURING UNDERSTANDING

INT-6

When a physical system involves an extended rigid body, there are two conditions of equilibrium—a translational condition and a rotational condition.

LEARNING OBJECTIVE

INT-6.A

a. Calculate the magnitude and direction of the torque associated with a given force acting on a rigid body system.

b. Calculate the torque acting on a rigid body due to the gravitational force.

ESSENTIAL KNOWLEDGE

INT-6.A.1

The definition of torque is:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

a. Torque is a vector product (or cross-product), and it has a direction that can be determined by the vector product or by applying the appropriate right-hand rule.

b. The idea of the "moment-arm" is useful when computing torque. The moment arm is the perpendicular distance between the pivot point and the line of action of the point of application of the force. The magnitude of the torque vector is equivalent to the product of the moment arm and the force.

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-6.B a. Describe the two conditions of equilibrium for an extended rigid body. b. Calculate unknown magnitudes and directions of forces acting on an extended rigid body that is in a state of translational and rotational equilibrium.	**INT-6.B.1** The two conditions of equilibrium are: a. $\sum \vec{F} = 0$ b. $\sum \vec{\tau} = 0$ c. Both conditions must be satisfied for an extended rigid body to be in equilibrium.
INT-6.C a. Explain the differences in the moments of inertia between different objects such as rings, discs, spheres, or other regular shapes by applying the general definition of moment of inertia (rotational inertia) of a rigid body. b. Calculate by what factor an object's rotational inertia will change when a dimension of the object is changed by some factor. c. Calculate the moment of inertia of point masses that are located in a plane about an axis perpendicular to the plane.	**INT-6.C.1** The general definition of moment of inertia is: $I = \sum m_i r_i^2$

continued on next page

LEARNING OBJECTIVE

INT-6.D

a. Derive the moment of inertia, using calculus, of a thin rod of uniform density about an arbitrary axis perpendicular to the rod.

b. Derive the moment of inertia, using calculus, of a thin rod of nonuniform density about an arbitrary axis perpendicular to the rod.

c. Derive the moments of inertia for a thin cylindrical shell or disc about its axis or an object that can be considered to be made up of coaxial shells (e.g., annular ring).

ESSENTIAL KNOWLEDGE

INT-6.D.1

The calculus definition of moment of inertia is:

$$I = \int r^2 dm$$

a. The differential dm must be determined from the linear mass density of the rod or object.

LEARNING OBJECTIVE

INT-6.E

Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.

ESSENTIAL KNOWLEDGE

INT-6.E.1

The parallel axis theorem is a simple powerful theorem that allows the moments of inertia to be computed for an object through any axis that is parallel to an axis through its center of mass.

$$I' = I_{cm} + Md^2$$

TOPIC 5.2

Rotational Kinematics

Required Course Content

ENDURING UNDERSTANDING

CHA-4

There are relationships among the physical properties of angular velocity, angular position, and angular acceleration.

LEARNING OBJECTIVE

CHA-4.A

a. Explain how the angular kinematic relationships for uniform angular acceleration are directly analogous to the relationships for uniformly and linearly accelerated motion.

b. Calculate unknown quantities such as angular positions, displacement, angular speeds, or angular acceleration of a rigid body in uniformly accelerated motion, given initial conditions.

c. Calculate unknown quantities such as angular positions, displacement, angular velocity, or rotational kinetic energy of a rigid body rotating with a specified nonuniform angular acceleration.

ESSENTIAL KNOWLEDGE

CHA-4.A.1

There are angular kinematic relationships for objects experiencing a uniform angular acceleration. These are the relationships:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

Other relationships can be derived from the above two relationships.

a. The appropriate unit for angular position is radians.

b. The general calculus kinematic linear relationships have analogous representations in rotational motion such as:

$$\omega = \frac{d\theta}{dt}$$

continued on next page

SUGGESTED SKILLS

Question and Method

2.B Make a claim or predict the results of an experiment.

Theoretical Relationships

5.B Determine the relationship between variables within an equation when an existing variable changes.

Mathematical Routines

6.C Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

LEARNING OBJECTIVE

CHA-4.B

a. Explain the use of the relationships that connect linear translational motion to rotational motion in appropriate physical situations.

b. Calculate the translational kinematic quantities from an object's rotational kinematic quantities for objects that are rolling without slipping.

c. Calculate the (tangential) linear acceleration of a point on a rotating object given the object's angular acceleration.

ESSENTIAL KNOWLEDGE

CHA-4.B.1

For objects that are rolling without slipping on a surface, the angular motion is related to the linear translational motion by the following relationships:

$v = r\omega$

$a = r\alpha$

$\Delta x = r\Delta\theta$

TOPIC 5.3

Rotational Dynamics and Energy

Required Course Content

ENDURING UNDERSTANDING

INT-7

A net torque acting on a rigid extended body will produce rotational motion about a fixed axis.

LEARNING OBJECTIVE

INT-7.A

a. Describe the complete analogy between fixed axis rotation and linear translation for an object subject to a net torque.

b. Calculate unknown quantities such as net torque, angular acceleration, or moment of inertia for a rigid body undergoing rotational acceleration.

c. Calculate the angular acceleration of an extended rigid body, of known moment of inertia, about a fixed axis or about its center of mass when it is experiencing a specified net torque due to one or several applied forces.

ESSENTIAL KNOWLEDGE

INT-7.A.1

The rotational analog to Newton's second law is:

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I}$$

a. In the appropriate cases, both laws (Newton's second law and the analogous rotational law) can be applied to a dynamic system and the two laws are independent from each other.

continued on next page

SUGGESTED SKILLS

Visual Representations

1.E Describe the effects of modifying conditions or features of a representation of a physical situation.

Representing Data and Phenomena

3.C Sketch a graph that shows a functional relationship between two quantities.

Data Analysis

4.D Select relevant features of a graph to describe a physical situation or solve problems.

Theoretical Relationships

5.D Determine or estimate the change in a quantity using a mathematical relationship.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

LEARNING OBJECTIVE

INT-7.B

a. Describe the net torque experienced by a rigid extended body in situations such as, but not limited to, rolling down inclines, pulled along horizontal surfaces by external forces, a pulley system (with rotational inertia), simple pendulums, physical pendulums, and rotating bars.

b. Derive an expression for all torques acting on a rigid body in various physical situations using Newton's second law of rotation.

ESSENTIAL KNOWLEDGE

INT-7.B.1

All real forces acting on an extended rigid body can be represented by a rigid body diagram. The point of application of each force can be indicated in the diagram.

a. The rigid body diagram is helpful in applying the rotational Newton's second law to a rotating body.

LEARNING OBJECTIVE

INT-7.C

Derive expressions for physical systems such as Atwood machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are—

a. rolling (or rotating on a fixed axis) without slipping.

b. rotating and sliding simultaneously.

ESSENTIAL KNOWLEDGE

INT-7.C.1

A complete analysis of a dynamic system that is rolling without slipping can be performed by applying both of Newton's second laws properly to the system.

a. The rotational characteristics may be related to the linear motion characteristics with the relationships listed in section CHA-4.A1 and CHA-4.B.1 (i.e., $v = r\omega$)

b. If the rigid body undergoing motion has a rotational component of motion and an independent translational motion (i.e., the object is slipping), then the rolling condition relationships do not hold.

$$v \neq r\omega$$

continued on next page

LEARNING OBJECTIVE

INT-7.D

a. Calculate the rotational kinetic energy of a rotating rigid body.

b. Calculate the total kinetic energy of a rolling body or a body that has both translation and rotational motion.

c. Calculate the amount of work done on a rotating rigid body by a specified force applied to the rigid body over a specified angular displacement.

ESSENTIAL KNOWLEDGE

INT-7.D.1

The definition of rotational kinetic energy is:

$$K_R = \frac{1}{2} I \omega^2$$

a. Total kinetic energy of a rolling body or a body with both forms of motion is the sum of each kinetic energy term.

b. The definition of work also has an analogous form in rotational dynamics:

$$W = \int \tau \, d\theta$$

LEARNING OBJECTIVE

INT-7.E

Derive expressions using energy conservation principles for physical systems such as rolling bodies on inclines, Atwood machines, pendulums, physical pendulums, and systems with massive pulleys that relate linear or angular motion characteristics to initial conditions (such as height or position) or properties of rolling body (such as moment of inertia or mass).

ESSENTIAL KNOWLEDGE

INT-7.E.1

If a rigid body is defined as "rolling," this implies (in the ideal case) that the frictional force does no work on the rolling object. The consequence of this property is that in some special cases (such as a sphere rolling down an inclined surface), the conservation of mechanical energy can be applied to the system.

SUGGESTED SKILLS

Visual Representations

1.E Describe the effects of modifying conditions or features of a representation of a physical situation.

Theoretical Relationships

5.E Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

Mathematical Routines

6.D Assess the reasonableness of results or solutions.

Argumentation

7.D Provide reasoning to justify a claim using physical principles or laws.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 5.4
Angular Momentum and Its Conservation

Required Course Content

ENDURING UNDERSTANDING

CON-5

In the absence of an external torque, the total angular momentum of a system can transfer from one object to another within the system without changing the total angular momentum of the system.

LEARNING OBJECTIVE

CON-5.A

a. Calculate the angular impulse acting on a rotating rigid body given specified angular properties or forces acting over time intervals.

b. Calculate the angular momentum vector of a rotating rigid body in cases in which the vector is parallel to the angular velocity vector.

ESSENTIAL KNOWLEDGE

CON-5.A.1

The definition of angular momentum of a rotating rigid body is:

$$\vec{L} = I\vec{\omega}$$

a. Angular impulse is equivalent to the change in angular momentum. The definition of this relationship is:

$$\int \vec{\tau}\, dt = \Delta\vec{L}$$

b. The differential definition is:

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
CON-5.B Calculate the angular momentum vector of a linearly translating particle about a defined stationary point of reference.	**CON-5.B.1** The angular momentum of a linearly translating particle can be defined about some arbitrary point of reference or origin. The definition is: $\vec{L} = \vec{r} \times \vec{p}$ a. The direction of this particle's angular momentum is determined by the vector product (cross-product).
CON-5.C a. Describe the conditions under which a rotating system's angular momentum is conserved. b. Explain how a one- or two-particle system (rotating object or satellite orbits) may have a change in angular velocity when other properties of the system change (such as radius or inertia).	**CON-5.C.1** In the absence of external torques acting on a rotating body or system, the total angular momentum of the system is a constant.
CON-5.D a. Calculate changes in angular velocity of a rotating rigid body when the moment of inertia of the body changes during the motion (such as a satellite in orbit). b. Calculate the increase or decrease in angular momentum of a rigid body when a point mass particle has a collision with the rigid body. c. Calculate the changes of angular momentum of each disc in a rotating system of two rotating discs that collide with each other inelastically about a common rotational axis.	**CON-5.D.1** The conservation of angular momentum can be applied to many types of physical situations. In all cases, it must be determined that there is no net external torque on the system. a. In the case of collisions (such as two discs colliding with each other), the torques applied to each disc are "internal" if the system is considered to be the two discs. b. In the case of a particle colliding with a rod or physical pendulum, the system is considered to be the particle and the rod together.

AP PHYSICS C: MECHANICS

UNIT 6

Oscillations

6–14%
AP EXAM WEIGHTING

~5/~10
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

Personal Progress Check 6

Multiple-Choice: ~10 questions
Free-Response: 1 question

UNIT 6

6–14% AP EXAM WEIGHTING

~5/~10 CLASS PERIODS

Oscillations

BIG IDEA 2 ACT

- How does the presence of restoring forces predict and lead to harmonic motion?

Developing Understanding

While earlier units focused on linear motion, Unit 6 pays close attention to the type of motion we experience when we talk or listen to music. Through the concept of oscillations, students are introduced to the model of simple harmonic motion (SHM), springs, and pendulums. Students will discover why some objects repeat their motions with a regular pattern. They will also apply the model of SHM, define the three kinematic characteristics (displacement, velocity, and acceleration), and practice representing them graphically and mathematically. During their study of oscillations, students will gain a more in-depth understanding of motion, making them better equipped to apply their knowledge of forces and motion to waves. Students will continue to expand on circular motion in Unit 7 as they explore celestial bodies and objects.

Building the Science Practices

4.C 4.E 7.F

Linearizing data is an important skill that helps students to re-express data, see functional relationships, and write equations between variables. Regular analysis of data collected in the laboratory, as well as laboratory type data, will strengthen students' linearization and data analysis skills.

Representations exist to make describing, explaining, and analyzing phenomena more manageable. It is vital that students are given multiple opportunities to represent data to be able to discuss how that data or graph illustrates physics principles, processes, concepts, or theories.

Unit 6 will continue to develop the practice of writing clear and concise laboratory observations and data collection, as well as identifying and/or describing potential sources of experimental error. Students should have multiple opportunities every unit to perform laboratory investigations, particularly inquiry-based investigations where they are challenged to think about, discuss, and analyze the potential sources of error and how each error affects the results of the experiment.

Preparing for the AP Exam

One of the free-response questions may require students to create graphs, free-body and free-body force diagrams, or other representations. Students often fail to create effective graphs due to an inability to choose appropriate quantities and labels (with units), a useful scale, correctly plot given data points, and draw a best-fit line. These elements must be performed correctly to receive full credit. Additionally, students must be adept at working with graphs (creation, analysis, interpreting, etc.) and constructing coherent arguments for them or other phenomena. For example, justifying the claim that an object is experiencing a restoring force based on the analysis of its motion and comparison to SHM.

Some students also struggle with correctly drawing free-body diagrams. It's recommended that students have ample practice and opportunities to practice this skill. For example, when drawing free-body force diagrams, students should pay careful attention to including appropriate forces and correct relative vector lengths.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~5/~10 CLASS PERIODS
INT-8	**6.1 Simple Harmonic Motion, Springs, and Pendulums**	**1.E** Describe the effects of modifying conditions or features of a representation of a physical situation. **2.B** Make a claim or predict the results of an experiment. **2.F** Explain modifications to an experimental procedure that will alter results. **4.C** Linearize data and/or determine a best fit line or curve. **4.E** Explain how the data or graph illustrates a physics principle, process, concept, or theory. **5.E** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway. **7.F** Explain how potential sources of experimental error may affect results and/or conclusions.	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 6.
Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 115 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	6.1	**Changing Representations** Students are given a graph of position/velocity/acceleration for SHM and must make the other three graphs with the same time scale, along with force, momentum, kinetic energy, potential energy, and total energy versus time graphs. The students must also make energy bar charts for various instants during the SHM.
2	6.1	**Desktop Experiment** Obtain a steel ruler or yardstick, clamp it to a table, and attach various masses to the end with the hole in it. Have students measure the period of oscillation for each mass attached, and then use the data to determine the spring constant of the steel ruler.
3	6.1	**Desktop Experiment** Have students use a pendulum to determine the acceleration of gravity in the classroom. The winners are the group whose procedure includes the most components for reducing error (timing multiple periods, linearizing data, very precisely finding the center of mass of the bob, for example).
4	6.1	**Ranking** Give students four to six cases of a mass on a spring. The cases show different masses, spring constants, and oscillation amplitudes (m/k/2A, m/2k/A, and 2m/k/2A, for example). Have students rank them based on period, frequency, maximum speed, maximum acceleration, maximum force, and total energy.
5	6.1	**Predict and Explain** Have students predict whether a ball rolling back and forth inside a spherical bowl is SHM. Have them take data to show whether this is SHM (period independent of amplitude or motion is a sine function or force proportional to displacement).

Unit Planning Notes

Use the space below to plan your approach to the unit.

SUGGESTED SKILLS

Visual Representations

1.E Describe the effects of modifying conditions or features of a representation of a physical situation.

Question and Method

2.B Make a claim or predict the results of an experiment.

2.F Explain modifications to an experimental procedure that will alter results.

Data Analysis

4.C Linearize data and/or determine a best fit line or curve.

4.E Explain how the data or graph illustrates a physics principle, process, concept, or theory.

Theoretical Relationships

5.E Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

Argumentation

7.F Explain how potential sources of experimental error may affect results and/or conclusions.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 6.1
Simple Harmonic Motion, Springs, and Pendulums

Required Course Content

ENDURING UNDERSTANDING

INT-8

There are certain types of forces that cause objects to repeat their motions with a regular pattern.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-8.A a. Describe the general behavior of a spring-mass system in SHM in qualitative terms. b. Describe the relationship between the phase angle and amplitude in an SHM system.	**INT-8.A.1** The general relationship for SHM is given by the following relationship: $x = x_{\max}\cos(\omega t + \varphi)$ φ is the phase angle and $x_{\max}$ is the amplitude of the oscillation. This expression can be simplified given initial conditions of the system.
INT-8.B a. Describe the displacement in relation to time for a mass-spring system in SHM. b. Identify the period, frequency, and amplitude of the SHM in a mass-spring system from the features of a plot.	**INT-8.B.1** The period of SHM is related to the angular frequency by the following relationship: $T = \frac{2\pi}{\omega} = \frac{1}{f}$

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-8.C Describe each of the three kinematic characteristics of a spring-mass system in SHM in relation to time (displacement, velocity, and acceleration). For a spring-mass system in SHM— a. describe the general features of the motion and b. identify the places on a graph where these values are zero or have maximum positive values or maximum negative values.	**INT-8.C.1** Using calculus and the position in relation to time relationship for an object in SHM, all three kinematic characteristics can be explored. Recognizing the positions or times where the trigonometric functions have extrema or zeroes can provide more detail in qualitatively describing the behavior of the motion.
INT-8.D Derive a differential equation to describe Newton's second law for a spring-mass system in SHM or for the simple pendulum.	**INT-8.D.1** Using Newton's second law, the following characteristic differential equation of SHM can be derived: $\frac{d^2x}{dt^2} = -\omega^2 x$ The physical characteristics of the spring-mass system (or pendulum) can be determined from the differential relationship.
INT-8.E Calculate the position, velocity, or acceleration of a spring-mass system in SHM at any point in time or at any known position from the initial conditions and known spring constant and mass.	**INT-8.E.1** All of the characteristics of motion in SHM can be determined by using the general relationship $x = x_{max} \cos(\omega t + \varphi)$ and calculus relationships.

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
INT-8.F Derive the expression for the period of oscillation for various physical systems oscillating in SHM.	**INT-8.F.1** The period can be derived from the characteristic differential equation. The following types of SHM systems can be explored: a. Mass oscillating on spring in vertical orientation b. Mass oscillating on spring in horizontal orientation c. Mass-spring system with springs in series or parallel d. Simple pendulum e. Physical pendulum f. Torsional pendulum
INT-8.G Calculate the mechanical energy of an oscillating system. Show that this energy is conserved in an ideal SHM spring-mass system.	**INT-8.G.1** Potential energy can be calculated using the spring constant and the displacement from equilibrium of a mass-spring system: $U_s = \frac{1}{2}k(\Delta x)^2$ a. Mechanical energy is always conserved in an ideal oscillating spring-mass system. b. Maximum potential energy occurs at maximum displacement, where velocity is zero and kinetic energy is zero. This maximum potential energy is equivalent to the total mechanical energy of the system. c. These energy relationships are true in the following three types of SHM systems: i. Mass-spring in horizontal orientation ii. Mass-spring in vertical orientation iii. Simple pendulum
INT-8.H Describe the effects of changing the amplitude of a spring-mass system.	**INT-8.H.1** Total energy of a spring-mass system is proportional to the square of the amplitude. $E_{total} = \frac{1}{2}kA^2 = \frac{1}{2}kx^2_{\max}$ a. The total energy is composed of the two contributing mechanical energies of the spring-mass system. $E_{total} = K + U_s$

continued on next page

LEARNING OBJECTIVE

INT-8.I

Describe the kinetic energy as a function of time (or position), potential energy as a function of time (or position), and total mechanical energy as a function of time (or position) for a spring-mass system in SHM, identifying important features of the oscillating system and where these features occur.

ESSENTIAL KNOWLEDGE

INT-8.I.1

The total mechanical energy of a system in SHM is conserved. The potential energy of the spring-mass system is:

$$U_s = \frac{1}{2}k(\Delta x)^2$$

and the kinetic energy of the system is:

$$K = \frac{1}{2}mv^2$$

The total energy in the system is defined above in **INT-8.H.1**.

INT-8.J

Explain how the model of SHM can be used to determine characteristics of motion for other physical systems that can exhibit this behavior.

INT-8.J.1

Any physical system that creates a linear restoring force ($\vec{F}_{rest} = -k\Delta\vec{x}$) will exhibit the characteristics of SHM.

INT-8.K

Describe a linear relationship between the period of a system oscillating in SHM and physical constants of the system.

INT-8.K.1

The period of a system oscillating in SHM is:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

(or its equivalent for a pendulum or physical pendulum) and this can be shown to be true experimentally from a plot of the appropriate data.

$$T_p = 2\pi\sqrt{\frac{l}{g}}$$

AP PHYSICS C: MECHANICS

UNIT 7

Gravitation

6–14%
AP EXAM WEIGHTING

~5/~10
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

Personal Progress Check 7

Multiple-Choice: ~10 questions
Free-Response: 1 question

UNIT 7

6–14% AP EXAM WEIGHTING | ~5/~10 CLASS PERIODS

Gravitation

Developing Understanding

Unit 7 investigates Newton's laws of gravity and the relationships shared between planets, satellites, and their orbits. Students will become familiar with the law of universal gravitation and how it can be applied to any pair of masses and will consider the motion of an object in orbit under the influence of gravitational forces. Additionally, students will be given opportunities to relate connected knowledge across units by applying and deriving Kepler's laws of planetary motion to circular or general orbits. Drawing such relationships will help elevate students' understanding of motion and force in various circumstances.

BIG IDEA 3
Fields **FLD**

- How does the moon stay in orbit despite its great distance from the Earth?

BIG IDEA 4
Conservation **CON**

- Why is navigation technology dependent on the orbits of Earth's artificial satellites?

Building the Science Practices

5.E 1.D 3.D

At this point in the course, students should be skilled at crafting clear and concise derivations that utilize fundamental principles of physics and follow a clear and logical algebraic pathway. Students should also be able to assess the feasibility of their solutions by checking units and functional relationships to determine if their solution is correct. This is an especially important skill when performing derivations, as the solution is often something that looks "messy" and is difficult to assess without practice.

As students progress, they should become more proficient at selecting and creating relevant representations (e.g., free-body diagrams) to answer a question or defend claims, illustrate physical situations, solve problems, and show consistency between multiple sets of representations of the same physical scenario.

Preparing for the AP Exam

Students should be able to assess the reasonableness of solutions. For gravity-based problems, they must be able to assess whether 500 kg is a reasonable solution for the mass of the sun or the mass of a person. They should also be able to construct a logical and coherent argument (or aspects of one) that supports their assessment or any other phenomena. Aspects or elements of an argument include explanations, predictions, and justifications.

Teachers should provide ample time and practice for students to develop and/or evaluate an argument using scientific reasoning that connects their claim and evidence; assumptions and limitations should also be considered. For example, students should be able to develop an explanation that supports the assertion that there exists in the space between two massive bodies a point where the net gravitational force on a test object is zero. Student explanations must always include a claim supported by evidence and reasoning.

UNIT 7

Gravitation

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~5/~10 CLASS PERIODS
FLD-1	**7.1 Gravitational Forces**	**3.D** Create appropriate diagrams to represent physical situations. **4.E** Explain how the data or graph illustrates a physics principle, process, concept, or theory. **5.E** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.	
CON-6	**7.2 Orbits of Planets and Satellites**	**3.C** Sketch a graph that shows a functional relationship between two quantities. **5.D** Determine or estimate the change in a quantity using a mathematical relationship. **6.C** Calculate an unknown quantity with units from known quantities by selecting and following a logical computational pathway. **7.F** Explain how potential sources of experimental error may affect results and/or conclusions.	

Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 7. Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 115 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	7.1	**Identify Subtasks** Have students research the structure of the Earth (specifically the density and depth of the various layers of the Earth: crust, mantle, outer core, inner core) and then calculate what the gravitational field strength must be at the boundary of each layer.
2	7.1	**Predict and Explain** Have students predict whether an object dropped into a hole drilled into a uniformly-dense, non-rotating planet exhibits simple harmonic motion. Have students show that it does (because the gravitational force is proportional to displacement from the center).
3	7.2	**Bar Chart** Have students create an energy bar-chart for an actual comet or asteroid that orbits the sun. Next, have them research the orbital parameters of the asteroid to make to-scale bar charts. The perihelion should be between 20% and 70% of the aphelion.
4	7.2	**Desktop Experiment** Have students use the My Solar System PhET applet to establish a circular orbit of a planet whose mass is very small compared to the central star. Trying various combinations of radius, speed, star mass, and planet mass (always making a circular orbit), have students show evidence of Newton's Law of Universal Gravitation.
5	7.2	**Desktop Experiment** Have students use the My Solar System PhET applet to establish a circular orbit of a planet whose mass is very small compared to the central star. Trying various combinations of radius and speed (always making a circular orbit), have students show evidence of Kepler's Third Law.

Unit Planning Notes

Use the space below to plan your approach to the unit.

SUGGESTED SKILLS

Representing Data and Phenomena

3.D Create appropriate diagrams to represent physical situations.

Data Analysis

4.E Explain how the data or graph illustrates a physics principle, process, concept, or theory.

Theoretical Relationships

5.E Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 7.1
Gravitational Forces

Required Course Content

ENDURING UNDERSTANDING

FLD-1

Objects of large mass will cause gravitational fields that create an interaction at a distance with other objects with mass.

LEARNING OBJECTIVE

FLD-1.A

Calculate the magnitude of the gravitational force between two large spherically symmetrical masses.

ESSENTIAL KNOWLEDGE

FLD-1.A.1

The magnitude of the gravitational force between two masses can be determined by using Newton's universal law of gravitation.

$$\left|\vec{F}_G\right| = \frac{Gm_1m_2}{r^2}$$

LEARNING OBJECTIVE

FLD-1.B

Calculate the value for *g* or gravitational acceleration on the surface of the Earth (or some other large planetary object) and at other points outside of the Earth.

ESSENTIAL KNOWLEDGE

FLD-1.B.1

Using Newton's laws it can be shown that the value for gravitational acceleration at the surface of the Earth is:

$$g = \frac{GM_e}{R_e^2}$$

and if the point of interest is located far from the earth's surface, then *g* becomes:

$$g = \frac{GM_e}{r^2}$$

continued on next page

LEARNING OBJECTIVE

FLD-1.C

Describe the motion in a qualitative way of an object under the influence of a variable gravitational force, such as in the case where an object falls toward the Earth's surface when dropped from distances much larger than the Earth's radius.

ESSENTIAL KNOWLEDGE

FLD-1.C.1

The gravitational force is proportional to the inverse of distance squared; therefore, the acceleration of an object under the influence of this type of force will be nonuniform.

SUGGESTED SKILLS

Representing Data and Phenomena

3.C Sketch a graph that shows a functional relationship between two quantities.

Theoretical Relationships

5.D Determine or estimate the change in a quantity using a mathematical relationship.

Mathematical Routines

6.C Calculate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.

Argumentation

7.F Explain how potential sources of experimental error may affect results and/or conclusions.

AVAILABLE RESOURCES

Classroom Resources >

- **AP Physics Featured Question: Projectile Concepts**
- **AP Physics Featured Question: Raft with Hanging Weights**
- **Critical Thinking Questions in Physics**
- **Physics Instruction Using Video Analysis Technology**
- **Quantitative Skills in the AP Sciences**
- **Teaching Strategies for Limited Class Time**

TOPIC 7.2
Orbits of Planets and Satellites

Required Course Content

ENDURING UNDERSTANDING

CON-6

Angular momentum and total mechanical energy will not change for a satellite in an orbit.

LEARNING OBJECTIVE

CON-6.A

Calculate quantitative properties (such as period, speed, radius of orbit) of a satellite in circular orbit around a planetary object.

ESSENTIAL KNOWLEDGE

CON-6.A.1

The centripetal force acting on a satellite is provided by the gravitational force between satellite and planet.

a. The velocity of a satellite in circular orbit is inversely proportional to the square root of the radius and is independent of the satellite's mass.

LEARNING OBJECTIVE

CON-6.B

Derive Kepler's third law for the case of circular orbits.

ESSENTIAL KNOWLEDGE

CON-6.B.1

In a circular orbit, Newton's second law analysis can be applied to the satellite to determine the orbital velocity relationship for satellite of mass *m* about a central body of mass *M*.

a. With proper substitutions, this can be reduced to expressing the period's dependence on orbital distance as Kepler's third law shows:

$$T^2 = \frac{4\pi^2}{GM} r^3$$

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
CON-6.C Describe a linear relationship to verify Kepler's third law.	**CON-6.C.1** Verifying Kepler's third law with actual data provides experimental verification of the law.
CON-6.D Calculate the gravitational potential energy and the kinetic energy of a satellite/Earth system in which the satellite is in circular orbit around the earth.	**CON-6.D.1** The gravitational potential energy of a satellite/Earth system (or other planetary/satellite system) in orbit is defined by the potential energy function of the system: $U_g = -\frac{Gm_e m_{sat}}{r}$ a. The kinetic energy of a satellite in circular orbit can be reduced to an expression that is only dependent on the satellite's system and position.
CON-6.E Derive the relationship of total mechanical energy of a satellite/earth system as a function of radial position.	**CON-6.E.1** The total mechanical energy of a satellite is inversely proportional to the orbital distance and is always a negative value and equal to one half of the gravitational potential energy.
CON-6.F a. Derive an expression for the escape speed of a satellite using energy principles. b. Describe the motion of a satellite launched straight up (or propelled toward the planet) from the planet's surface, using energy principles.	**CON-6.F.1** In ideal situations, the energy in a planet/satellite system is a constant. a. The gravitational potential energy of a planet/satellite system is defined to have a zero value when the satellite is at an infinite distance (very large planetary distance) away from the planet. b. By definition, the "escape speed" is the minimum speed required to escape the gravitational field of the planet. This could occur at a minimum when the satellite reaches a nominal speed of approximately zero at some very large distance away from the planet.

continued on next page

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
CON-6.G Calculate positions, speeds, or energies of a satellite launched straight up from the planet's surface, or a satellite that is projected straight toward the planet's surface, using energy principles.	**CON-6.G.1** In ideal nonorbiting cases, a satellite's physical characteristics of motion can be determined using the conservation of energy.
CON-6.H Describe elliptical satellite orbits using Kepler's three laws of planetary motion.	**CON-6.H.1** The derivation of Kepler's third law is only required for a satellite in a circular orbit.
CON-6.I a. Calculate the orbital distances and velocities of a satellite in elliptical orbit using the conservation of angular momentum. b. Calculate the speeds of a satellite in elliptical orbit at the two extremes of the elliptical orbit (perihelion and aphelion).	**CON-6.I.1** In all cases of orbiting satellites, the total angular momentum of the satellite is a constant. a. The conservation of mechanical energy and the conservation of angular momentum can both be used to determine speeds at different positions in the elliptical orbit.