AP Physics C: Mechanics
Unit 3 – Analysis of Circular Motion and Orbits
Assessment

Don't run around in circles trying to solve these, although they might make your head spin!

Name: ____ Period: __________

SUMMARY

- An observer in a non-inertial (accelerating) reference frame introduces fictitious forces when applying Newton's Laws within that frame; observers outside and observing the same will not need to do this. If the fictitious forces are properly defined, the description of motion in the non-inertial frame will be the equivalent to that made by an observer in an inertial frame. However, observers in the two frames will not agree on what causes the motion.
- Centrifugal force is a fictitious force.
- A particle moving in uniform circular motion has centripetal acceleration. This acceleration must be provided by a net force toward the center of the circle defining the path.
- Newton's Second law applied to a particle moving in uniform circular motion states that the net force in the radial direction must equal the product of the mass and centripetal acceleration:

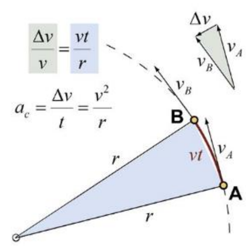
Centripetal Force Equation

Combining these two equations . . .

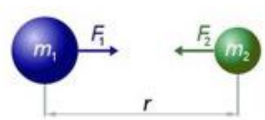
$F_c = ma_c$ and $a_c = \frac{v^2}{r}$

you get:

$$F_c = \frac{mv^2}{r}$$



- Newton's Universal Law of Gravitation states:



$$F_1 = F_2 = G\frac{m_1 m_2}{r^2}$$

$$G \cong 6.674 \times 10^{-11} \frac{m^3}{kg - s^2}$$

- For a satellite in Earth's orbit at an altitude of h (m), we find

$$\therefore \frac{mV_c^2}{r} = G\frac{M.m}{r^2}$$

$$\therefore V_c^2 = \frac{GM}{r}$$

$$\therefore Vc = \sqrt{\frac{GM}{R+h}} \quad \because r = R + h$$

- For a satellite in geosynchronous orbit, its period must be 24 hours (86,400 s).

CONCEPTS

1) Earth is a non-inertial frame for two reasons: 1) it rotates on its axis, and 2) it orbits the Sun. Let us assume that Earth's orbit is a circle and that Earth is a uniform sphere. How would the apparent weight of an object compare between the poles and the equator? Does the mass of the object change?
2) Earth is not a perfect sphere – its equatorial diameter is slightly larger than its polar diameter. Why?
3) Why does an astronaut aboard the International Space Station experience "weightlessness?"
4) An artist is molding clay on her flywheel. Yay. Why would the clay fly off the wheel if she spins it too fast?
5) Jack and Jill are returning from the hill carrying a pail of water. Jack, being a gentleman, is carrying the pail. Jack, also being an idiot, decides to swing the pail in a vertical circle. However, even at the top, water does not fall on his wee head. Explain.
6) In the lab, Phil Physicson attaches a 10 kg mass to the end of a spring and then whirls the spring/mass system over his head by holding the free end of the spring. Would the spring stretch? Why or why not?
7) NASA and JPL scientists have proposed that future space stations actually be large rotating cylinders 16 km long and 8 km in diameter. This space station would rotate along its long axis, thus simulating gravity on the outer walls. How do you explain this artificial gravity? Is the gravity real or fictitious?
8) Is there a situation where a car can have centripetal acceleration but no tangential acceleration?
9) Is there a situation where a car can have tangential acceleration but no centripetal acceleration?

PROBLEMS

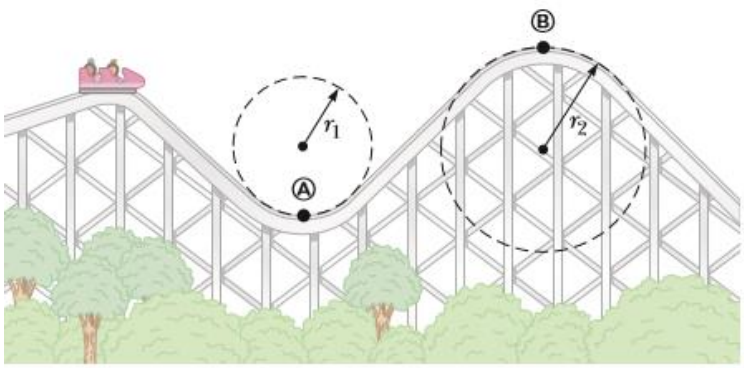
10) It is found that a certain string (L = 0.8 m) of negligible mass can support a mass of 25 kg before it breaks. On a frictionless horizontal table, one end of this string is secured to motor; at the other end is a 3 kg mass. What range of speeds can the mass have before breaking?
11) During the American Apollo missions, one astronaut would stay in orbit 100 km above the Moon while two other astronauts travelled to the surface. Given that the mass of the Moon is 7.4×10^{22} kg and its radius is 1.7×10^{6} m, determine
    a. the acceleration of the orbiting astronaut,
    b. the astronaut's orbital speed, and
    c. the period of the orbit.
12) A car is driving with a constant speed with its cruise control turned on. The car drives over a hill with a radius of 18 m. At the top of the hill, the driver feels a brief sensation of weightlessness; that is, like she is barely in contact with the driver's seat. What is the speed of the vehicle?
13) Mr. Webber enjoys listening to his old Barry Manilow records to unwind after a day of dealing with AP Physics students. However, his old record player sometimes needs additional weight put on it to spin properly. During a particularly moving rendition of "Copacabana," Mr. Webber had to place a coin (all that is left over from his paycheck) 30 cm from the center of the rotating record to get it working properly. However, as Mr. Webber dances around showing off his sweet disco moves, Mrs. Webber sneaks in and switches the machine to spin at 50 cm/s, causing the coin to slip. The video Mrs. Webber took of Mr. Webber disco dancing to that spin rate is available for a small fee.
    a. What provides the centripetal force when the coin is stationary relative to the record?
    b. What is the coefficient of static friction between the coin and the record?
14) Mr. Webber is out riding his motorcycle, wearing his leather jacket that has his biker name stitched on the back ("Big Daddy Bunny"). He approaches a bump in the road at 9.0 m/s. The radius of this bump is 11.0 m. If a passenger on the motorcycle has a weight of 600 N…
    a. …what apparent weight does the passenger feel going over the bump?
    b. …at what speed would Mr. Webber have to drive the motorcycle so that the passenger experiences weightlessness?

15) The fair is in town, and AP Physics students are out enjoying the rides, cotton candy, and salty fried food that Mr. Webber adds additional salt to. The Ferris wheel at this fair has a radius of 20 m and makes one revolution every 9.0 s. If one of the AP students has a mass of 55 kg, what force does she exert on the seat when she is at the top of the Ferris wheel?

16) It's Physics Day at Busch Gardens! The brave and daring DASOTA AP Physics students decide to ride the roller coaster. The roller coaster's car has a mass of 500 kg when completely loaded with passengers. Consider the diagram of the roller coaster below:

a. What is the force of the track on the vehicle at point A (r_1 = 10 m) if the car has a speed at that point of 20 m/s?

b. What is the maximum speed the car can have at point B (r_2 = 15 m) so that it remains on the track?



17) George of the Jungle is swinging on a vine trying to cross a river full of the dreaded cottenheaded-water rattle snakes. The vine is 10 m long and George has a mass of 85 kg. At the bottom of the swing, where George would just barely clear the surface, his speed is 8 m/s. His pal, an ape named "Ape" (really. Watch the cartoon.), knows that the vine has a breaking point of 1000 N. Does George make it safely across the ravine?

18) A 45 kg child is sitting on the floor of a horizontal merry-go-round 3.0 m from the center. His friends push on the ride, accelerating it to a constant speed of one revolution every 12 s.

a. What is the acceleration of the child sitting on the merry-go-round when it reaches its constant speed?

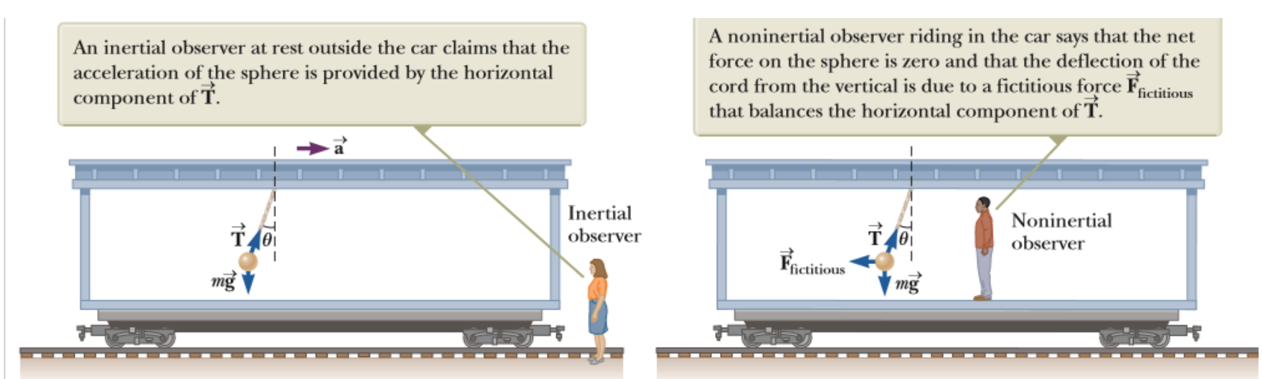
b. Find the frictional force acting on the child.

c. What is the minimum coefficient of static friction necessary to keep the child from slipping?

19) A 0.5 kg object is suspended from the ceiling of an accelerating boxcar, as shown below. If the acceleration of the boxcar is 3.0 m/s^2, find

a. the angle that the string makes with the vertical, and

b. the tension in the string.



20) Meadowlark Lemmon, the great basketball player from the Harlem Globetrotters, is spinning a ball of diameter 10.0 cm on his finger. It slows uniformly from 30 rev/min to rest in 0.3 s. For a point on the equator of the ball at the beginning of this time period, calculate

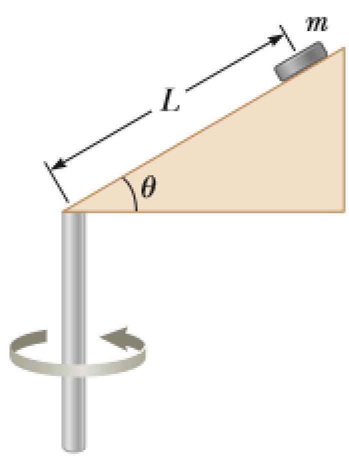
a. the radial acceleration.
b. tangential acceleration.
c. net acceleration.

21) As we have discussed, because Earth rotates on its axis, a point on the equator experiences a centripetal acceleration of 0.034 m/s^2, while points directly at the poles experience no centripetal acceleration. You may assume Earth is a uniform sphere and g = 9.800 m/s^2.

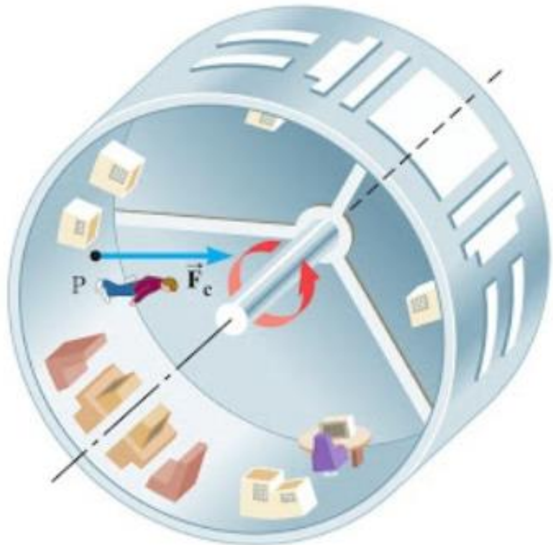
a. Show that at the equator the gravitational force on an object (the true weight: $\boldsymbol{W} = m\boldsymbol{g}$) must exceed the objects apparent weight.
b. Consider a person who has a mass of 75 kg. What is the apparent weight of this person at the equator and at the poles?

22) Before there was the Internet, television, or electricity (you guys wouldn't have survived!), there was a popular child's toy that consisted of a wedge and a mass that was on top of the frictionless slope. The mass would stay in place as the toy was spun around a rotating rod, which is firmly attached to the wedge at one end (below).

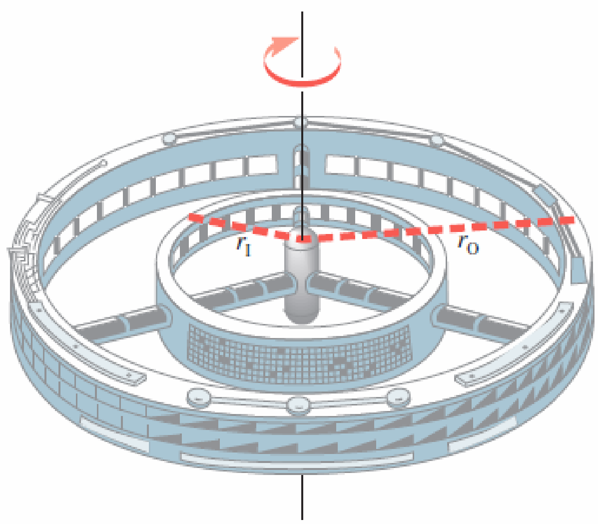
a. Since friction is negligible, what force is keeping the mass on the slope?
b. Derive an equation of motion that satisfies $v = f(g, L, \theta)$.



23) Astronauts face many physiological challenges in a weightless environment. To minimize these effects, artificial gravity will be used in space stations of the future, as shown below. Because of rotational motion, any object located at a point P on the interior surface of the station experiences a centripetal force toward the axis. The surface of the station provides the artificial gravity by applying a force on the object. If the space station depicted above has a radius of 1700 m, at what speed must the interior surface of the space station move so that an astronaut standing at point P experiences a push on his feet that equals his weight on Earth?



24) A space laboratory is rotating to create artificial gravity, as shown. Its period of rotation is chosen so that the outer ring (r_o = 2150 m) simulates the acceleration due to gravity on Earth (9.8 m/s^2). What should the radius r_1 of the inner ring be so that it simulates the acceleration due to gravity on the surface of Mars (3.72 m/s^2)?



25) To create artificial gravity, the space station shown in the drawing below is rotating at a rate of 1.0 rpm. The radii of the cylindrically-shaped chambers have the ratio $r_A/r_B = 4.0$. Each chamber A simulates an acceleration due to gravity of 10 m/s^2. Find values for

a. r_A,

b. r_B

c. the acceleration due to gravity that is simulated in chamber B.