NATIONAL MATH + SCIENCE INITIATIVE

What I Absolutely Have to Know to Survive the AP* Exam

Kinematics is the study of *how* things move – how far (*distance* and *displacement*), how fast (*speed* and *velocity*), and how fast that *how fast* changes (*acceleration*). We say that an object moving in a straight line is moving in *one dimension*, and an object which is moving in a curved path (like a *projectile*) is moving in *two dimensions*. We relate all these quantities with a set of equations called the *kinematic equations*.

Two-dimensional motion includes objects which are moving in two directions at the same time, such as a *projectile*, which has both horizontal and vertical motion. These two motions of a projectile are completely independent of one another, and can be described by *constant velocity* in the horizontal direction, and *free fall* in the vertical direction. Since the two-dimensional motion described in this packet involves only constant acceleration, we may use the *kinematic equations*. If an object moves in the horizontal and vertical direction at the same time, we say that the object is moving in *two dimensions*. We subscript any quantity which is horizontal with an x (such as v_x and a_x), and we subscript any quantity which is vertical with a y (such as v_y and a_y.)

Key Formulas and Relationships

$$a = \frac{v - v_o}{t}$$

$$v = v_o + at$$

$$\Delta x = \frac{1}{2}(v_o + v)t$$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

Where Δx = displacement (final position – initial position)
v = velocity or speed at any time
v_o = initial velocity or speed
t = time
a = acceleration

Horizontal direction:

$$v_x = v_{ox} + a_x t$$

$$x = \frac{1}{2}(v_{ox} + v_x)t$$

$$x = v_{ox}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{ox}^2 + 2a_x x$$

Vertical direction:

$$v_y = v_{oy} + a_y t$$

$$y = \frac{1}{2}(v_{oy} + v_y)t$$

$$y = v_{oy}t + \frac{1}{2}a_y t^2$$

$$v_y^2 = v_{oy}^2 + 2a_y y$$

For a projectile near the surface of the earth: $a_x = 0$, v_x is constant, and $a_y = g = 10$ m/s^2.

acceleration
the rate of change in velocity

acceleration due to gravity
the acceleration of a freely falling object in the absence of air resistance, which near the earth's surface is approximately 10 m/s^2

acceleration-time graph
plot of the acceleration of an object as a function of time

average acceleration
the acceleration of an object measured over a time interval

average velocity
the velocity of an object measured over a time interval; the displacement of an object divided by the change in time during the motion

constant (or uniform) acceleration
acceleration which does not change during a time interval

constant (or uniform) velocity
velocity which does not change during a time interval

displacement
change in position in a particular direction (vector)

distance
the length moved between two points (scalar)

free fall
motion under the influence of gravity

initial velocity
the velocity at which an object starts at the beginning of a time interval

instantaneous
the value of a quantity at a particular instant of time, such as instantaneous position, velocity, or acceleration

kinematics
the study of how motion occurs, including distance, displacement, speed, velocity, acceleration, and time

position-time graph
the graph of the motion of an object that shows how its position varies with time

speed
the ratio of distance to time

velocity
ratio of the displacement of an object to a time interval

projectile
any object that is projected by a force and continues to move by its own inertia

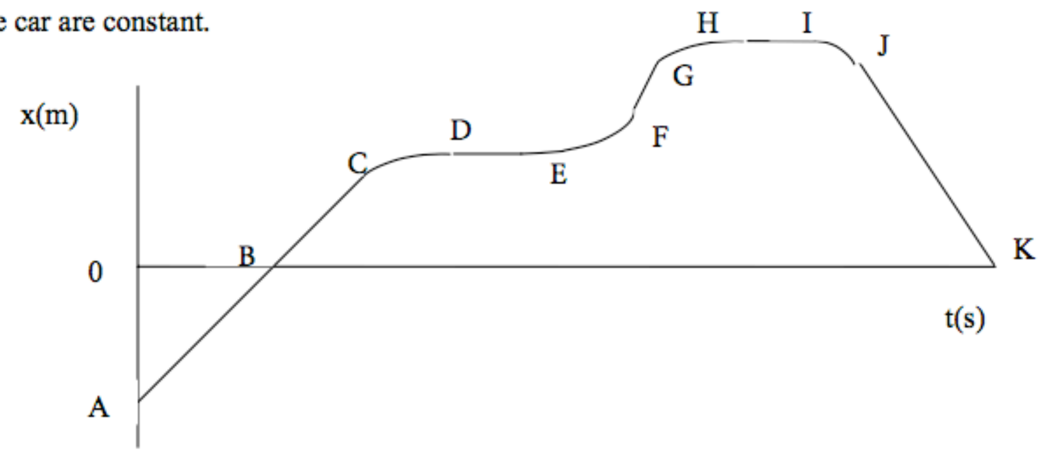
range of a projectile
the horizontal distance between the launch point of a projectile and where it returns to its launch height

trajectory
the path followed by a projectile

DISCUSSION OF SELECTED SECTIONS		
Displacement	ΔX or ΔY	*Distance d* can be defined as total length moved. If you run around a circular track, you have covered a distance equal to the circumference of the track. Distance is a scalar, which means it has no direction associated with it. *Displacement* $\Delta \mathbf{x}$, however, is a vector. Displacement is defined as the straight-line distance between two points, and is a vector which points from an object's initial position $\mathbf{x_0}$ toward its final position $\mathbf{x_f}$. In our previous example, if you run around a circular track and end up at the same place you started, your displacement is zero, since there is no distance between your starting point and your ending point. Displacement is often written in its scalar form as simply Δx or x.
Speed and Velocity	v	Average speed is defined as the amount of distance a moving object covers divided by the amount of time it takes to cover that distance: average speed = $v = \frac{\text{distance}}{\text{elapsed time}} = \frac{d}{t}$ where v stands for speed, d is for distance, and t is time. *Average velocity* is defined a little differently than *average speed*. While average speed is the total change in distance divided by the total change in time, average velocity is the *displacement* divided by the change in time. Since velocity is a vector, we must define it in terms of another vector, displacement. Oftentimes average speed and average velocity are interchangeable for the purposes of the AP Physics 1 exam. Speed is the magnitude of velocity, that is, speed is a scalar and velocity is a vector. For example, if you are driving west at 50 miles per hour, we say that your speed is 50 mph, and your velocity is 50 mph west. We will use the letter v for both speed and velocity in our calculations, and will take the direction of velocity into account when necessary.
Acceleration	a	Acceleration tells us how fast velocity is changing. For example, if you start from rest on the goal line of a football field, and begin walking up to a speed of 1 m/s for the first second, then up to 2 m/s for the second second, then up to 3 m/s for the third second, you are speeding up with an average acceleration of 1 m/s for each second you are walking. We write $a = \frac{\Delta v}{\Delta t} = \frac{1 m/s}{1 s} = 1 m/s/s = 1 \frac{m}{s^2}$

Example 1

Consider the *position vs. time* graph below representing the motion of a car. Assume that all accelerations of the car are constant.

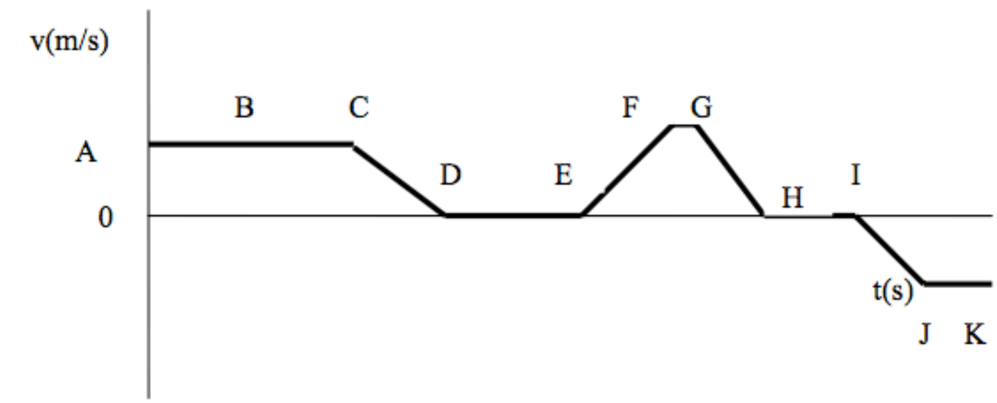


On the axes below, sketch the *velocity vs. time* and *acceleration vs. time* graphs for this car.

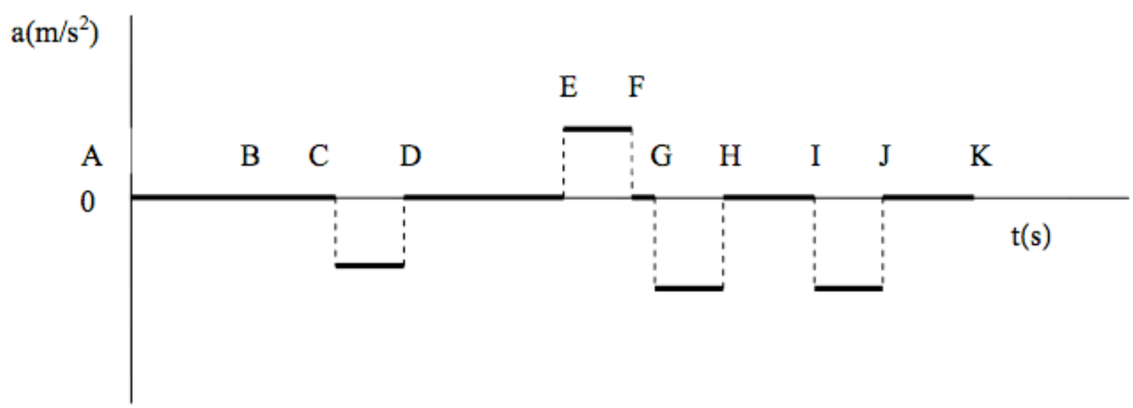
Solution:

The car starts out at a distance behind our reference point of zero, indicated on the graph as a negative displacement. The velocity (slope) of the car is initially positive and constant from points A to C, with the car crossing the reference point at B. Between points C and D, the car goes from a high positive velocity (slope) to a low velocity, eventually coming to rest ($v = 0$) at point D. At point E the car accelerates positively from rest up to a positive constant velocity from points F to G. Then the velocity (slope) decreases from points G to H, indicating the car is slowing down. It is between these two points that the car's velocity is positive, but its acceleration is negative, since the car's velocity and acceleration are in opposite directions. The car once again comes to rest at point H, and then begins gaining a negative velocity (moving backward) from rest at point I, increasing its speed negatively to a constant negative velocity between points J and K. At K, the car has returned to its original starting position.

The *velocity vs. time* graph for this car would look like this:



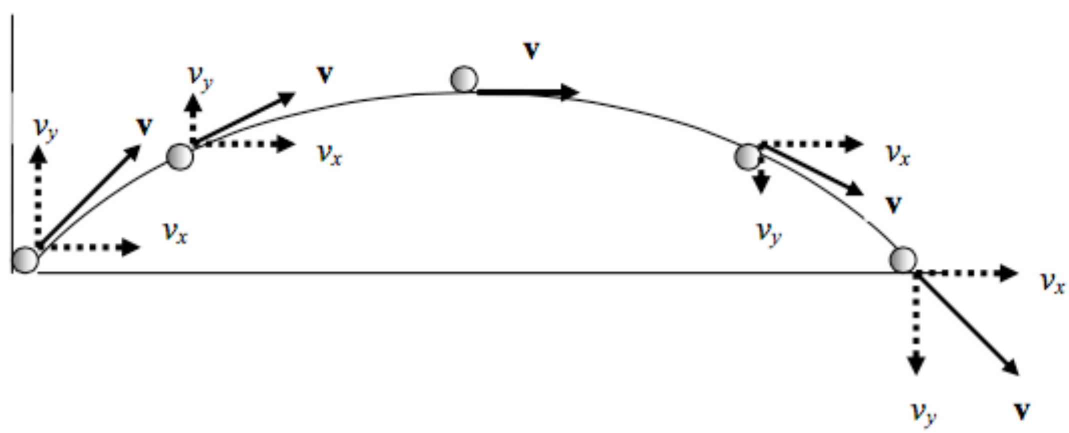
The *acceleration vs. time* graph for this car would look like this:



Projectile Motion

Projectile motion results when an object is thrown either horizontally through the air or at an angle relative to the ground. In both cases, the object moves through the air with a constant horizontal velocity, and at the same time is falling freely under the influence of gravity. In other words, the projected object is moving horizontally and vertically at the same time, and the resulting path of the projectile, called the *trajectory*, has a parabolic shape. For this reason, projectile motion is considered to be *two-dimensional* motion.

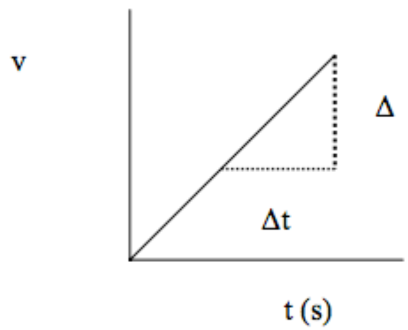
The motion of a projectile can be broken down into constant velocity and zero acceleration in the horizontal direction, and a changing vertical velocity due to the acceleration of gravity. Let's label any quantity in the horizontal direction with the subscript x, and any quantity in the vertical direction with the subscript y. If we fire a cannonball from a cannon on the ground pointing up at an angle θ, the ball will follow a parabolic path and we can draw the vectors associated with the motion at each point along the path:

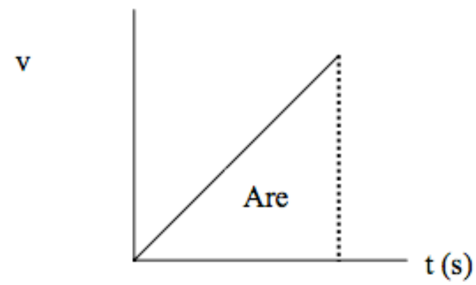


At each point, we can draw the horizontal velocity vector $\mathbf{v_x}$, the vertical velocity vector $\mathbf{v_y}$, and the vertical acceleration vector **g**, which is simply the acceleration due to gravity. Notice that the length of the horizontal velocity and the acceleration due to gravity vectors do not change, since they are constant. The vertical velocity decreases as the ball rises and increases as the ball falls. The motion of the ball is symmetric, that is, the velocities and acceleration of the ball on the way up are the same as on the way down, with the vertical velocity being zero at the top of the path and reversing its direction at this point.

Velocity vs. time

Consider the *velocity vs. time* graph below:

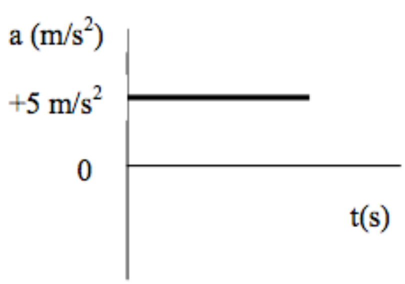


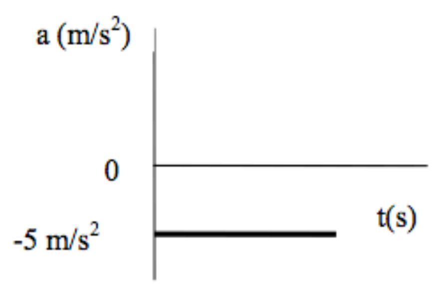


As shown in the figure on the left, the slope of a velocity vs. time graph is $\frac{\Delta v}{\Delta t}$, and is therefore acceleration. As shown on the figure on the right, the area under a velocity vs. time graph would have units of $\frac{m}{s}(s) = m$, and is therefore displacement.

Acceleration vs. time

Since the AP Physics 1 exam generally deals with constant acceleration, any graph of acceleration vs. time on the exam would likely be a straight horizontal line:





This graph on the left tells us that the acceleration of this object is positive. If the object were accelerating negatively, the horizontal line would be below the time axis, as shown in the graph on the right.