AP Physics C: Mechanics
Unit 4 – Work, Energy, and Power
Assessment

Do you have the energy to do this work?

Name: ____ Period: __________

Summary

- The **scalar, or dot, product** of any two vectors A and B is defined by the relationship

  $$\boldsymbol{A} \cdot \boldsymbol{B} \equiv AB \cos \theta$$

  where the result is a scalar quantity and θ is the included angle between the two vectors. The scalar product obeys the commutative and distributive laws.

- The **work** done by a constant force **F** acting on a particle is defined as the product of the component of the force in the direction of the particle's displacement and the magnitude of the displacement. It is therefore the dot product of force and displacement:

  $$W = \boldsymbol{F} \cdot \boldsymbol{d} = Fd \cos \theta$$

- The work done by a varying force acting on a particle moving along the x axis from x_i to x_f is given by:

  $$W \equiv \int_{x_i}^{x_f} F_x dx$$

  where F_x is the component of the force in the x direction. If there are several forces acting on a particle, the net work done by all forces is the sum of the individual work done by each force.

- Work and **energy** are closely related. Both are measured in Joules. Work is done only when energy transforms from one type to another; energy can only be measured by having it do work. Different observers in different reference frames may obtain different values for work and energy.
- The **kinetic energy, T,** of a particle of mass m moving with a speed v (where v is much smaller than the speed of light) is defined as:

  $$T \equiv \frac{1}{2}mv^2$$

- The **Work-Energy Theorem** states that the net work done on a particle by external forces is equal to the change in kinetic energy of the particle:

  $$W_{net} = T_f - T_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

- The **instantaneous power** is defined as the time rate of doing work. If an agent applies a force **F** to an object moving with a velocity **v**, the power delivered by that agent is given by

  $$P \equiv \frac{dW}{dt} = \boldsymbol{F} \cdot \boldsymbol{v}$$

- A force is **conservative** if the work done by that force acting on a particle is independent of the path the particle takes between two given points. Alternatively, a force is conservative if the work done by that force is zero when the particle moves through an arbitrary closed path and returns to its original position. A force that does not meet these criteria is said to be **nonconservative**.
- A potential energy function U can only be associated with a conservative force. If a conservative force F acts on a particle that moves along the x axis from x_i to x_f, the change in the potential energy equals the negative of the work done by that force:

$$U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

- The **Law of the Conservation of Mechanical Energy** states that if the only force acting on a mechanical system is conservative, then the total mechanical energy is conserved:

$$T_i + U_i = T_f + U_i$$

- The **total mechanical energy** of a system is defined as the sum of the kinetic energy and the potential energy:

$$E \equiv T + U$$

- The **gravitational potential energy** of a particle of mass m that is evaluated a distance y near Earth's surface is given by:

$$U_g \equiv mgy$$

- The **work-energy theorem (revised)** also states that the work done by all nonconservative forces acting on a system equals he change in the total mechanical energy of the system:

$$W_{nc} = E_f - E_i$$

- The **elastic potential energy** stored in a spring of force constant k is:

$$U_s \equiv \frac{1}{2}kx^2$$

<u>Concepts</u>

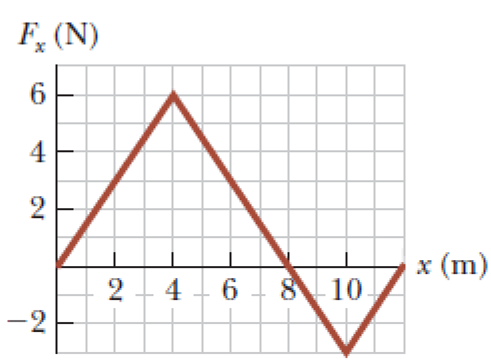
1) Can the kinetic energy of an object ever be negative?
2) If the speed of a particle is doubled, what happens to its kinetic energy?
3) What can be said about the speed of an object if the net work done on that object is zero?
4) A simple pendulum swings back and forth. For this analysis, the only forces acting on the pendulum are gravity, tension, and fluid friction (air resistance).
    a. Which of these forces, if any, do no work on the pendulum?
    b. Which of these forces does negative work on the pendulum at all times?
    c. Describe the work done by the force of gravity while the pendulum is swinging.
5) Can the gravitational potential energy of an object ever have a negative value? Explain.
6) A ball is dropped by a person from the top of a building, while another person at the bottom of the building observes the motion. Will these two people agree on the value of the ball's potential energy? The change in potential energy? On the kinetic energy?

7) When nonconservative forces act on a system, does the total mechanical energy remain constant? Explain.
8) A block is connected to a spring that is suspended from the ceiling. If the block is set in motion and air resistance is ignored, will the total energy of the system be conserved? How many forms of potential energy are there for this system?
9) Consider a ball fixed to one end of a rigid rod with the other end pivoted on a horizontal axis so that the rod can rotate in a vertical plane. What are the positions of stable and unstable equilibrium?

Problems

10) A force of 70 N acting at an angle of 20° with respect to the horizontal is pulling a 15 kg block over a rough, horizontal surface ($\mu = 0.3$). In a displacement of 5 m, find the work done by
    a. the 70 N force,
    b. the force of friction,
    c. the normal force, and
    d. the force of gravity.
    e. What is the net work done on the block?
11) Imagine that you have to push a 40 kg box of textbooks across a horizontal floor. The coefficient of kinetic friction between the box and the floor is 0.25. If you slide the box at a constant speed of 1.4 m/s, at what rate is work being done on the crate by
    a. you?
    b. the frictional force?
12) Consider a vector in space described by $\boldsymbol{A} = -2\boldsymbol{i} + 3\boldsymbol{j}$.
    a. Find the magnitude of $\boldsymbol{A}$.
    b. What angle does $\boldsymbol{A}$ make with the positive y axis?
13) A particle undergoes a displacement of $\boldsymbol{s} = (3\boldsymbol{i} + \boldsymbol{j})$ m under the influence of a force $\boldsymbol{F} = (6\boldsymbol{i} - 2\boldsymbol{j})$ N.
    a. Find the work done on the particle by the force; and,
    b. the angle between $\boldsymbol{F}$ and $\boldsymbol{s}$.
14) The force acting on some particle in space varies as shown in the graph below. Find the work done by the force as the particle moves
    a. from x = 0 m to x = 8 m,
    b. from x = 8 m to x = 10 m, and
    c. from x = 0 to x = 10 m.

    Note: This is a level 1 problem; that is, one more appropriate for AP 1, as an examination of the graph shows motion changing instantaneously. However, it does serve as good conceptual practice.



15) A Hooke's Law spring is exposed to a force that varies from 0 to 50 N as one end is stretched 12 cm from its unstressed position.
   a. Find the force constant k of the spring.
   b. Find the work done in stretching the spring.

16) An auto mechanic needs to push a disabled car (m = 2500 kg) from rest to some speed v. The mechanic does 5000 J in the process, moving the car 25 m. Assume that in this short distance that friction is negligible.
   a. What is the final speed of the car?
   b. What is the horizontal force exerted on the car?

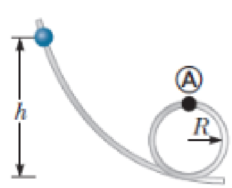
17) A bullet is fired into a tree, penetrating it a distance of 4 cm. If the bullet has a mass of 5.0 g and a velocity of 600 m/s,
   a. Use work and energy considerations to find the average frictional force that stops the bullet.
   b. Assuming that the frictional force is constant, determine the time that elapsed between when the bullet entered the tree to the moment it stopped.

18) There exists an inclined plane that makes a 20^{o} angle with the horizontal. At the bottom of this plane is a 4 kg block. The block is given an initial velocity of 8 m/s up the incline and experiences a frictional force of 15 N.
   a. How far will the block move up the ramp before it stops?
   b. Will it slide back down the incline? Why or why not?

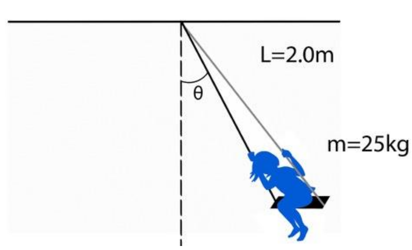
19) A car (m = 1500 kg) accelerates uniformly from rest to a speed of 10 m/s in 3 s.
   a. Find the work done on the car in this time.
   b. What is the average power delivered by the engine during the three seconds?
   c. What is the instantaneous power delivered by the engine at t = 2 s?

20) A conservative force, F_x = (2x + 4) N acts on a particle (m = 5 kg). As the particle moves from x = 1 m to x = 5 m, calculate
   a. the work done by the force;
   b. the change in potential energy of the particle; and,
   c. the kinetic energy of the particle at x = 5 m if the particle had a velocity of 3 m/s at x = 1 m.

21) A bead slides down a smooth loop-the-loop, as shown. If the bead is released from a height of h = 3.5R,
   a. what is the speed of the bead at point A?
   b. If the mass of the bead is 5.0 g, how large is the normal force at point A?

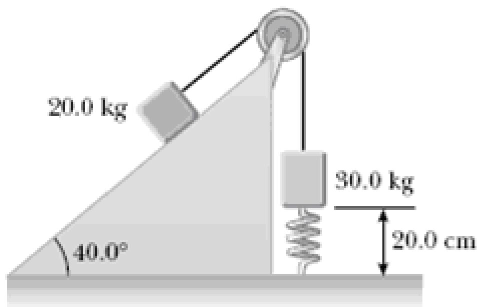


22) A child of mass 25 kg is playing on a swing that is 2 m long. Her friend pulls her back and releases her from rest when the swing supports make an angle of 30^{o} with the vertical.
   a. Neglecting friction, find the speed of the child at the lowest position.
   b. If the speed of the child at the lowest position is 2 m/s, what was the energy loss due to friction?



23) A crane is engineered to lift 2000 kg of material a height of 150.0 m in 1 minute at a uniform rate. What electrical power is required to drive the crane motor if only 35% of the electric power is converted to mechanical power?

24) A 30 kg block is connected to a 20 kg block by a light string that passes over a frictionless pulley. The 30 kg mass is attached to a light spring of force constant 250 N/m. The 20 kg mass is pulled a distance of 20 cm down the frictionless ramp (40^o) and is released from rest. Find the speed of each block when the 30 kg block is 20 cm above the floor (the spring's unstretched position).



25) A potential energy function for a two-dimensional force is of the form $U = 3x^3y - 7x$. Find the force that acts at the point (x, y).